Sectoral Technology Differences and the Labor Share

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Abstract. The labor share of the United States is lower today than in 1980 on every measure, and the price of consumption relative to investment rose over the same decades. The usual link between the two facts, substitution toward cheaper capital, requires an elasticity of substitution between capital and labor above one, while firm and industry estimates put it below one. This paper explores how a deviation from balanced growth, through a gap between the growth rates of technology across sectors, can explain the observed paths of the relative price and the labor share. In a two-sector growth model with CES technologies and competitive markets, the labor share is constant along a balanced growth path, at any elasticity of substitution. A balanced growth path is one on which the technology gap is constant. The gap is the ratio of the labor-augmenting technology levels of the investment and the consumption sector. Sectoral differences therefore act while the gap widens and through the level at which it settles. While it widens, the wage grows faster than the technology of the consumption sector, and with substitution below one the labor share rises. With technological progress calibrated to the observed relative price and the economy starting from its 1980 state, the model implies, under competitive markets, a rising labor share over 1980–2024 at every elasticity below one, the more the lower the elasticity, barely in the Cobb-Douglas case, and a falling share at the elasticity above one that the usual reading requires. The widening of the gap accounts for most of the change at every elasticity. Sectoral technology differences are a force against the measured decline, and they move the allocation of capital against its measured path. They do not explain the decline.

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1. Introduction

The labor share of income in the United States is lower today than in 1980 on every definition in use. How much lower depends on the definition. The Hodrick-Prescott trends of twelve measures built from the national accounts and the BLS productivity program fall by between 2.0 and 8.0 percentage points (pp) over 1980–2024. The private-sector gross measure, the reference measure of this paper, falls by 4.0 pp. The timing does not depend on the definition: the trends are flat to 2000, every one falls over 2000–2012, and the fall then stops or slows. Over the same decades the price of consumption relative to investment rose by 43 log points.

The literature connects the two facts, the fall of the labor share and the rise of the relative price of consumption to investment, through the technology that the relative price reveals. Greenwood *et al.* (1997) read the fall in the quality-adjusted price of equipment relative to consumption as the trace of technological change specific to the production of investment goods. Karabarbounis & Neiman (2014) take the next step: cheaper investment goods lower the cost of capital, firms substitute capital for labor, and the labor share falls. The step requires an elasticity of substitution between capital and labor above one, since a fall in the rental rate of capital lowers the labor share only if capital and labor are gross substitutes (Grossman & Oberfield, 2022). With an elasticity below one the same fall raises the labor share. Estimates from firm and industry data put the elasticity below one, between 0.4 and 0.6 in the survey of Chirinko (2008) and between 0.5 and 0.7 for the aggregate of United States manufacturing in Oberfield & Raval (2021). The cross-country data agree once the rental rate is measured as the growth model requires: Glover & Short (2020) estimate an elasticity near or below one from the same data as Karabarbounis & Neiman (2014). By the same argument, at those values the technological change that the relative price reveals raises the labor share and works against the decline.

This paper asks how large that countervailing force is: by how much the technological change that the relative price reveals raises the labor share when the elasticity of substitution is below one. The question is how technology differences between the sector that produces consumption goods and the sector that produces investment goods are transmitted to the labor share. It is asked in a general equilibrium model calibrated to the United States economy over 1980–2024. The setting is the growth model with investment-specific technological change of Greenwood *et al.* (1997), in its two-sector representation, with three changes. Each sector produces with a constant elasticity of substitution (CES) technology in capital and labor, with an elasticity different from one, the two technologies differ in their distribution parameters, and labor-augmenting technological change proceeds at a rate specific to each sector. With distribution parameters that differ, the investment good is no longer produced with the technology of the consumption good scaled by a productivity factor. With an elasticity different from one, a difference in the technology levels removes that representation as well. The ratio of the two technology levels is the technology gap, and

the difference between the two rates of technological progress is its growth rate. Preferences are of the Greenwood-Hercowitz-Huffman form (Greenwood *et al.*, 1988), so hours respond only to the current wage, not to wealth. Markets are competitive, so there are no profits, and the model isolates what technology alone does. The path of the growth rate of the gap is fitted to the relative price and to nothing else. The other parameters are set from external estimates and from base-year data. The economy starts from its 1980 state, with the capital-output ratio of the data, which is not a balanced growth path. The simulated change therefore combines the convergence from that state with the effect of the widening gap. A path on which the gap stays at its 1980 level, the constant-gap path, is reported alongside and separates the two. The labor share is not targeted. It is read against the band of the twelve measures, so that a model path is judged within the range of the measured changes and not against one series.

Three results follow. The first is a general property of the model. Along a balanced growth path, on which the two technologies grow at the same rate, the relative price, the sector labor shares, and the aggregate labor share are constant, at any elasticity of substitution. A change in the labor share can therefore be explained only along a transition: sectoral technology differences move the share while the technology gap widens, and through the level of the gap at which the transition ends. The variable behind the mechanism is the wage per efficiency unit of labor in the consumption sector. Faster technology growth in the investment sector raises the wage faster than the technology of the consumption sector, so the cost of labor per efficiency unit rises there. With substitution below one, the share of labor rises with its cost. A second channel is composition: capital moves toward the investment sector, whose labor share is the higher of the two in 1980, and the aggregate rises with the weight of that sector. In the Cobb-Douglas case the first channel is closed and only the second remains.

The second result is quantitative. At the baseline elasticity of 0.5, a growth rate of the gap that falls over time reproduces the path of the relative price. Under that path the labor share of the model rises by 4.2 pp over 1980–2024. The private-sector gross measure falls by 4.0 pp over the same years, and the trends of the twelve measures by 2.0 to 8.0 pp. The two paths part after 2000, where the decline in the data is concentrated. The fit is as good at elasticities of 0.3, 0.7, 1, and 1.25, with the profile of the growth rate of the gap re-fitted at each value. The elasticity is therefore set from the estimates above. Their range maps into a rise of 2.3 to 6.6 pp. Under Cobb-Douglas technology the rise is 0.1 pp, the composition channel alone. At the cross-country estimate of 1.25 of Karabarbounis & Neiman (2014), the share falls by 1.3 pp, because the within-sector channel changes sign.

The third result concerns the variables that are not used in the calibration. The rise of the labor share in the model comes from the consumption sector, whose share rises by 5.4 pp over 1997–2024 while it falls by 4.1 pp in the data. Capital moves toward the investment sector in the model and away from it in the data. Hours per household rise by 19% in the

model and fall by 8% in the data, because the widening gap raises the wage in units of consumption faster than the trend to which the disutility of labor is tied. The rise in hours depends on that trend. With a trend that also grows with the gap, hours fall over the sample (Section 4.3). Output, consumption, capital, and the wage grow faster in the model than in the data over the period. The wage runs ahead from the start, output and consumption from the early 2000s, and capital from the late 2000s. Sectoral technology differences with an elasticity below one are therefore a countervailing force. At the baseline elasticity the rise of 4.2 pp is comparable in size to the 4.0 pp decline of the private-sector gross measure, the measure that matches the definition of the model's labor share. The same differences move the allocation of capital against its measured path.

The paper belongs to the literature that Grossman & Oberfield (2022) survey, and it touches four of its streams. The first is measurement. The choices behind the twelve measures are the ones the literature has debated: value added gross or net of depreciation (Bridgman, 2018; Rognlie, 2015), the capitalization of intellectual property products (Koh *et al.*, 2020), the labor component of proprietors' income (Elsby *et al.*, 2013), and the sectors covered. Karabarbounis (2024) surveys the measured declines across countries and industries. The paper takes no side among the measures. It treats their range as the measured change and reads the model inside it.

The second is the elasticity of substitution. The cross-country estimate above one of Karabarbounis & Neiman (2014) stands against the firm- and industry-level estimates below one (Chirinko, 2008; Oberfield & Raval, 2021), and the two readings of the same relative price have opposite signs. Glover & Short (2020) trace the cross-country estimate above one to the proxy for the rental rate. That proxy uses the investment price alone, where the Euler equation of the growth model also calls for consumption growth. Once the proxy is completed, they estimate an elasticity near or below one, with point estimates statistically indistinguishable from one. Lawrence (2015) shows that with an elasticity below one the decline can be read as the result of labor-augmenting technical change fast enough to lower the effective capital-labor ratio. Hubmer (2023) estimates gross substitutes from the relation between capital intensity and the change in the labor share across goods. At his estimates investment-specific technical change lowers the labor share, and he combines it with nonhomothetic demand, which raises the share as income grows. This paper takes the elasticity from the firm- and industry-level estimates, holds it below one in both sectors, and lets the relative price determine the path of the technology gap at each value.

The third is investment-specific technological change. The model is the two-sector representation of the growth model of Greenwood *et al.* (1997), read through the relative price of consumption to investment at the aggregate level. Its factor shares differ between the two sectors, as Valentinyi & Herrendorf (2008) and Basu *et al.* (2013) measure them at the level of final-use sectors. Both find the consumption sector the more capital-intensive. Two-sector models with CES technologies and elasticities below one have been used for

the same question. Díez-Catalán (2018) feeds the relative price of investment and sectoral capital- and labor-augmenting technical change into a model of services and non-services over 1980–2015. His aggregate compensation share falls by 4.6 pp, against 5.4 pp in the data, and the fall comes from the decline of capital-augmenting technology in non-services. Alvarez-Cuadrado *et al.* (2018) find that capital-biased technical change, combined with complementarity between capital and labor, drives the decline of the labor share in a two-sector model with sector-specific elasticities. The reallocation between sectors accounts for a small part of the measured change in their model. Alvarez-Cuadrado *et al.* (2017) show that differences in the substitutability of capital and labor across sectors drive structural change. Bergholt *et al.* (2022) find evidence of complementarity between capital and labor in United States time series, under which capital deepening raises the labor share. The model here has labor-augmenting technical change only, and its two sectors produce consumption and investment. In Díez-Catalán (2018) and Alvarez-Cuadrado *et al.* (2018) the decline comes from capital-augmenting or capital-biased technical change. This paper reads the relative price as a gap between the labor-augmenting technologies of the consumption and the investment sector. It fits the path of that gap to the relative price alone, at the elasticities of the firm- and industry-level estimates, and reads the untargeted labor share against the band of twelve measures. At those elasticities, with labor-augmenting change only, the widening gap raises the labor share.

The fourth is the stability of the labor share. Grossman & Oberfield (2022) observe that the share stayed within a narrow range for most of a century against the many forces able to move it, and suggest that the literature may have pursued mechanisms that predict a fall while setting aside those that predict a rise. The stabilizing forces they list operate in the long run, among them directed technical change (Acemoglu, 2002) and the schooling response of Grossman *et al.* (2017). Grossman *et al.* (2017) state the Uzawa restriction for falling investment-good prices. They show that a balanced growth path with capital-augmenting technical change exists when schooling is endogenous and capital is more complementary with schooling than with raw labor. On the balanced growth path of Grossman *et al.* (2021) the capital share falls with the rates of capital- and labor-augmenting technical change. In these models, Grossman & Oberfield (2022) note, investment-specific technical change lowers the capital share in the short run. In this paper the gap widens at a falling rate over the sample and the labor share rises. The force quantified here is of a different kind. It is an effect of the technology that the investment-specific reading already invokes, at the elasticities that the firm and industry estimates support. It acts along a transition and through the level of the gap at which the transition ends. The remaining mechanisms are left out by design: market power in product markets (Autor *et al.*, 2020; Barkai, 2020; De Loecker *et al.*, 2020), automation (Acemoglu & Restrepo, 2020), and the demographics and education that Grossman & Oberfield (2022) review. Grossman & Oberfield (2022) note that the amounts explained by the mechanisms, each estimated on its own, add up to three or four times the decline. Harrison (2024) estimates several of them jointly on firm records. This paper quantifies the effect of sectoral technology differences

on its own, in a model to which the other mechanisms can be added without changing the technology. The wedges that market power introduces between factor prices and marginal products are examined in Gouveia-Mendes (2026) within the same model.

The rest of the paper has four sections. Section 2 documents the two facts: the band of labor share measures and the relative price of consumption to investment. Section 3 sets out the model, reduces the equilibrium to a system in four variables, characterizes the balanced growth path, decomposes the labor share into its within-sector and composition channels, and states the Cobb-Douglas case. The proofs, and the case of unequal technology growth rates, are in the appendix. Section 4 calibrates the model, fits the path of the technology gap to the relative price, and reads the simulated labor share and the untargeted variables against the data. Section 5 concludes.

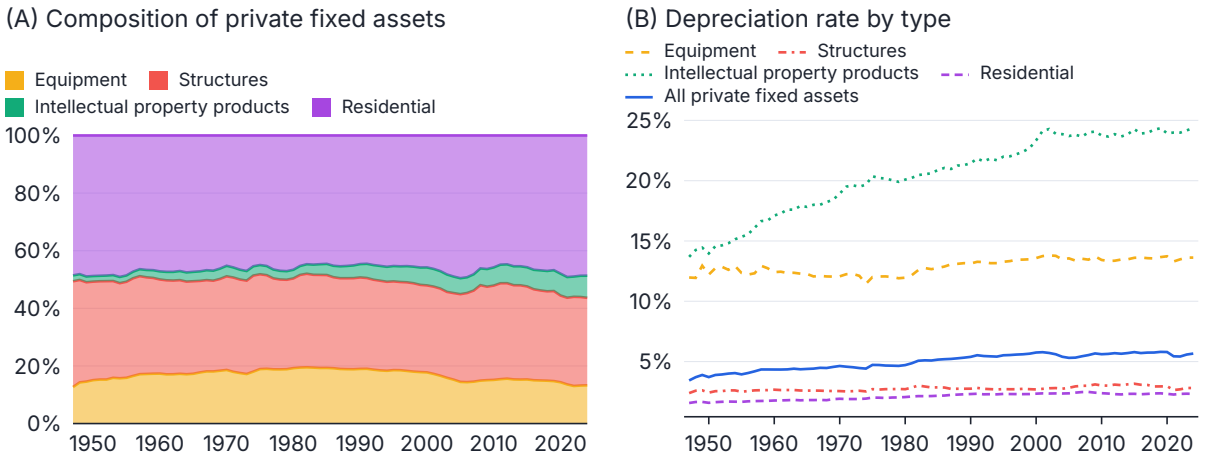
2. Facts

2.1. The Decline of the Labor Share

The labor share divides the compensation of workers by the value added they help produce. Neither term has a single definition, and the choices behind each one move the level of the series and the size of its change. Four of these choices account for most of the disagreement in the literature (Grossman & Oberfield, 2022).

The first choice is whether value added is measured gross or net of depreciation. Depreciation has grown as a share of output because the capital stock has shifted toward intellectual property, which wears out much faster than structures. A gross measure therefore records as capital income a growing flow that only replaces worn-out capital. This is why the gross share has left its historical range while the net share has not (Bridgman, 2018; Rognlie, 2015). The second choice concerns intellectual property products (IPP). The Bureau of Economic Analysis (BEA) treats them as investment, software since 1999 and research and development and artistic originals since 2013, whereas it had treated them as intermediate inputs before. The reclassification raises value added without raising compensation, so it lowers the measured share. Koh *et al.* (2020) show that a large part of the decline goes away when the old convention is restored. The third choice is the treatment of proprietors' income, which pays the self-employed for both their work and their capital. Allocating it entirely to capital understates the labor share, and the understatement changes over time as self-employment changes. Elsby *et al.* (2013) therefore impute a labor component on two bases. One applies the labor share of the rest of the economy, the other subtracts a market return on the assets of the self-employed. The last choice is coverage. The BLS headline series covers the nonfarm business sector, the BEA corporate series leave out the self-employed altogether, and the broader private-sector and whole-economy series add households, nonprofit institutions, and government, whose value added is mostly compensation by construction.

Figure 1. Private fixed assets: composition and depreciation, 1947–2024.



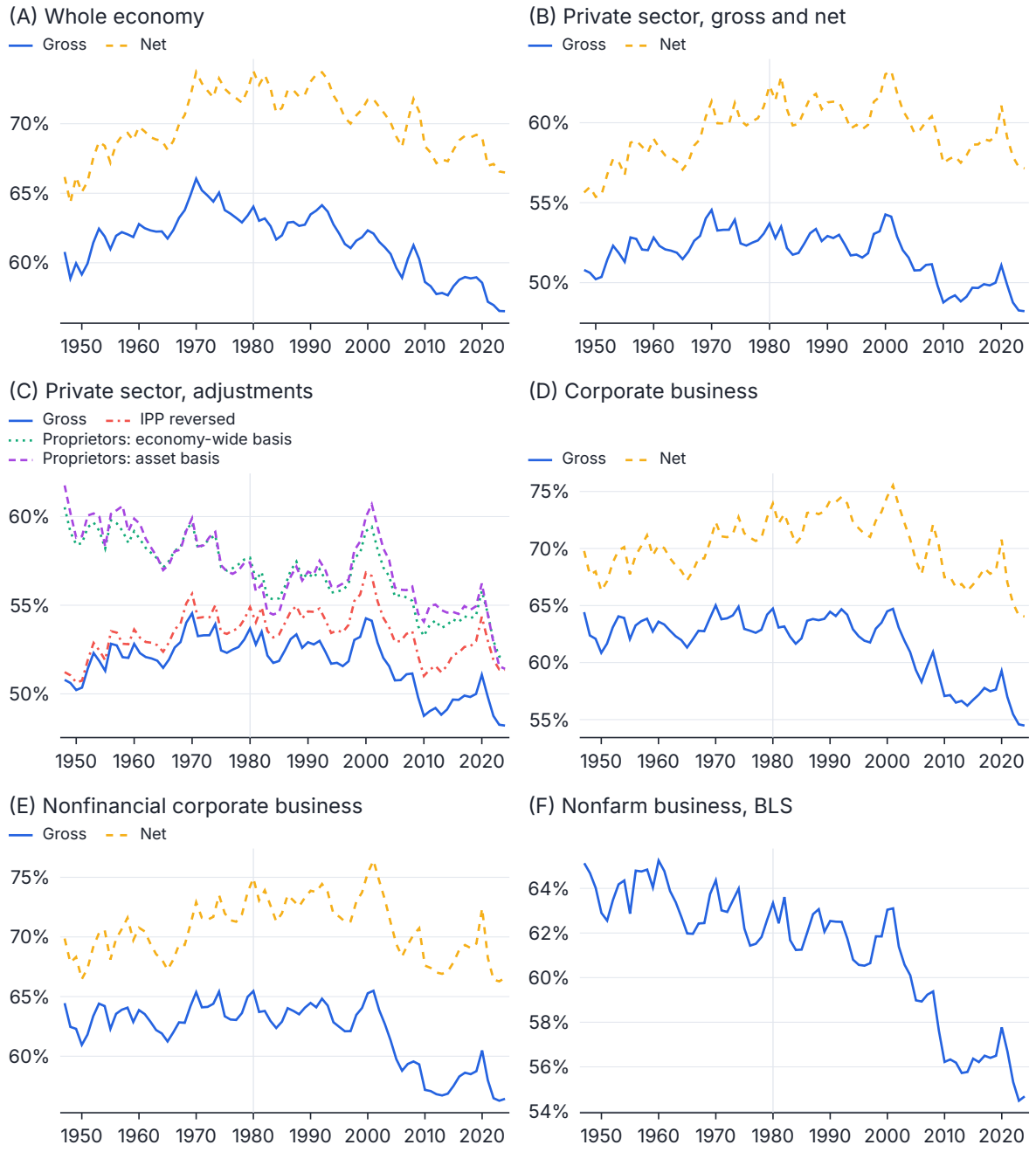
Notes: Annual, 1947–2024. (A) Share of each asset type in the current-cost net stock of private fixed assets (Fixed Assets Table 1.1). (B) Current-cost depreciation over the current-cost net stock, % per year, by type and for all private fixed assets (Fixed Assets Tables 1.3 and 1.1). Structures are nonresidential. Residential structures are the Residential series.

Two of the four choices, depreciation and IPP, trace back to the same change in the capital stock, and Figure 1 shows it. Panel (A) splits the current-cost net stock of private fixed assets into housing, nonresidential structures, equipment, and IPP. The share of IPP rises from 3.0% to 7.7% over 1980–2024. At current cost it is the only nonresidential share that grows: equipment loses 6 pp because its price falls faster than its quantity rises (Section 2.2). Panel (B) shows why the shift matters. IPP depreciates at 24% a year, equipment at 14%, nonresidential structures at 3%, and housing at 2%. The depreciation rate of the whole private stock rises from 4.7% to 5.7% a year. By a shift-share on the two panels, the composition of the stock accounts for 0.25 of the 0.96 pp, the rates within asset types for 0.60 pp, and their interaction for 0.11 pp. Through composition, the move toward IPP adds 0.94 pp and the fall in the equipment share takes away 0.71 pp. Private depreciation accordingly climbs from 13.5% to 15.2% of private value added. That flow is the wedge between the gross and the net labor share, and its growth is why the two measures tell different stories about the level of the share even when they agree on its fall.

Figure 2 follows these choices through twelve measures built from the BEA national accounts and the BLS productivity program. The whole-economy, private-sector, corporate, and nonfinancial corporate measures each come in a gross and a net version. The private sector adds the three adjustments to its gross measure: the IPP capitalization reversed, and proprietors' income imputed on each of the two bases. The BLS series is the twelfth. In 2024 the levels run from 48% to 67% of value added, so the definition moves the level by more than any single measure moved over the period.

The direction of change, by contrast, does not depend on the definition. Every one of the twelve measures is lower in 2024 than in 1980. To put a number on each decline without letting two particular years decide it, Table 1 compares the Hodrick-Prescott trend of each series in the two years, with the smoothing parameter set to 400, the value of Correia *et al.* (1992) for annual data. The size of the decline depends on the definition. The private-

Figure 2. Twelve measures of the U.S. labor share, 1947–2024.



Notes: % of value added, annual, 1947–2024. Vertical line: 1980, the base year of the changes. Panel (C) repeats the private-sector gross measure of panel (B) as the reference for the three adjustments. Private sector: compensation of employees of corporate and noncorporate business, households, and nonprofit institutions over the gross or net value added of the same sectors (NIPA Tables 1.3.5, 1.9.5, 1.13). IPP reversed: private IPP investment (Table 1.1.5) subtracted from value added. Proprietors' income: Table 1.10, allocated with the labor share of the rest of the private sector (economy-wide basis) or net of a return on noncorporate fixed assets at the corporate rate (asset basis, Fixed Assets Table 6.1). Whole economy: compensation over compensation plus net operating surplus of private enterprises and private depreciation (Table 1.10). Corporate and nonfinancial corporate: Table 1.14. BLS: nonfarm business labor share, index rescaled to the published 2017 level.

sector measures lose between 2.9 and 4.0 pp. Once the IPP capitalization is reversed the loss is only 2.0 pp, which confirms the weight of that convention. The whole-economy and nonfinancial corporate measures lose between 5.4 and 6.9 pp. The corporate measures and the BLS series lose between 6.7 and 8.0 pp, about twice the private-sector decline. Between the smallest and the largest decline there is a factor of about 4.

Table 1. Labor share measures: levels and changes, 1980–2024.

Measure	1980	2024	Change, HP trend	Change, raw
Private sector				
Gross	53.7	48.2	−4.0	−5.5
Net	62.3	57.1	−2.9	−5.2
Gross, IPP capitalization reversed	54.9	51.4	−2.0	−3.5
Gross, proprietors' income on the economy-wide basis	57.7	52.0	−4.0	−5.7
Gross, proprietors' income on the asset basis	57.4	51.4	−3.9	−6.0
Whole economy				
Gross	64.0	56.5	−6.6	−7.5
Net	73.8	66.5	−5.5	−7.3
Corporate business				
Gross	64.7	54.5	−8.0	−10.3
Net	73.9	64.0	−6.7	−9.9
Nonfinancial corporate business				
Gross	65.5	56.4	−6.9	−9.0
Net	74.9	66.6	−5.4	−8.4
Nonfarm business (BLS)				
Headline series	63.3	54.7	−7.3	−8.7

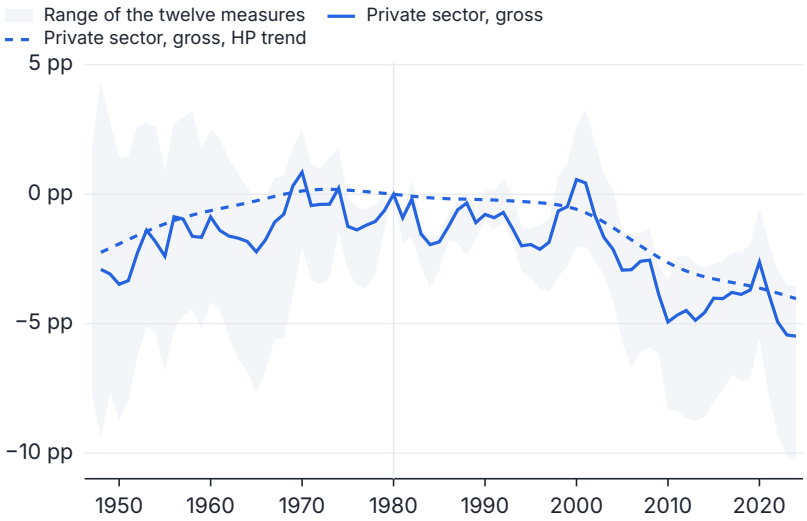
Notes: Levels in % of value added. Changes in percentage points, on the HP trend and raw. HP trend: the trend in 2024 minus the trend in 1980, fitted on each series' full sample (1947–2024, and 1948–2024 for the private-sector series) with smoothing parameter 400. Raw: the 2024 value minus the 1980 value, as plotted in Figure 3. With smoothing parameter 6.25, the annual value of Ravn & Uhlig (2002), the HP-trend changes run from −9.1 to −2.9 pp. With 100, the value of Backus & Kehoe (1992), they run from −8.0 to −2.0 pp. With 3-year averages at the two ends they run from −8.8 to −3.0 pp. Sources as in Figure 2.

On timing the measures agree again. Before 2000 the measures show no common decline. Over 1980–2000 the HP trends move between −1.9 and 1.1 pp, and the largest falls are those of the two whole-economy measures. The decline is concentrated in the following twelve years. Over 2000–2012 every trend falls, by 2.0 to 4.4 pp. Over 2012–2024 every trend falls again, by 0.3 to 3.0 pp, and all but one by less than over 2000–2012. For the private-sector gross measure the three phases contribute −0.6, −2.4, and −1.1 pp.

The shaded area of Figure 3 is the band of the twelve measures: in each year, the lowest and the highest change since 1980 across them, in percentage points. Inside it runs the private-sector gross measure, with its HP trend. It is the broadest of the measures that leave out government. A model path inside the band lies within the range of the measured changes.

The labor share is lower today than in 1980 on every definition. How much lower, between 2.0 and 8.0 pp, depends on the definition. The loss is concentrated in 2000–2012. Grossman & Oberfield (2022) reach the same reading: the measures disagree on how far the share has fallen, but they agree that it sits well below its level of the 1970s and 1980s.

Figure 3. The band of labor share measures: change since 1980.



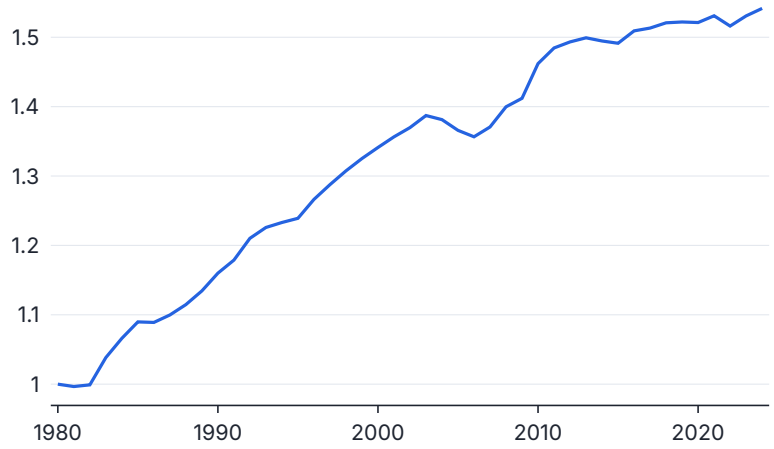
Notes: Each series minus its 1980 value, in percentage points. Shaded area: from the lowest to the highest value across the twelve measures in each year. Solid line: the private-sector gross measure. Vertical line: 1980, the base year of the changes. Dashed line: the HP trend of the private-sector gross measure, minus its 1980 value (smoothing parameter 400). Sources as in Figure 2.

2.2. The Decline of the Relative Price of Investment

This paper asks what part of that decline can be attributed to technology, in the form of productivity growth that differs between the sector producing consumption goods and the sector producing investment goods. As mentioned before, the ratio of the two technology levels is called the technology gap in this paper, and the difference between the two growth rates is its growth rate. Productivity growth is not observed directly. The model of Section 3 relates the equilibrium relative price of investment goods to the technology gap between the two sectors, and that relation is how the path of the gap is calibrated. The relative price of consumption to investment is the ratio of the price index of personal consumption expenditures to the price index of gross private domestic investment (NIPA Table 1.1.4, lines 2 and 7). Greenwood *et al.* (1997) read the postwar fall in the quality-adjusted price of equipment relative to consumption as investment-specific technological change. The relative price used here is broader. It covers all of private investment, including structures and residential investment, whose prices rise relative to consumption (Figure 5). Karabarbounis & Neiman (2014) document the rise of the relative price of consumption to investment across countries alongside the decline of the labor share. The behavior of the relative price over the period of the labor share decline is therefore the second fact to establish.

Figure 4 shows the relative price as an index with 1980 = 1. It rises by 43 log points over 1980–2024, to 1.54 in 2024. The rise slows down over the period. It runs at 1.5% per year over 1980–1997, 1.0% per year over 1997–2012, and 0.3% per year over 2012–2024. The two facts therefore differ in timing. The relative price moved fastest in the two decades over which the labor share was flat, and had almost stopped moving by the time the labor share settled at its lower level.

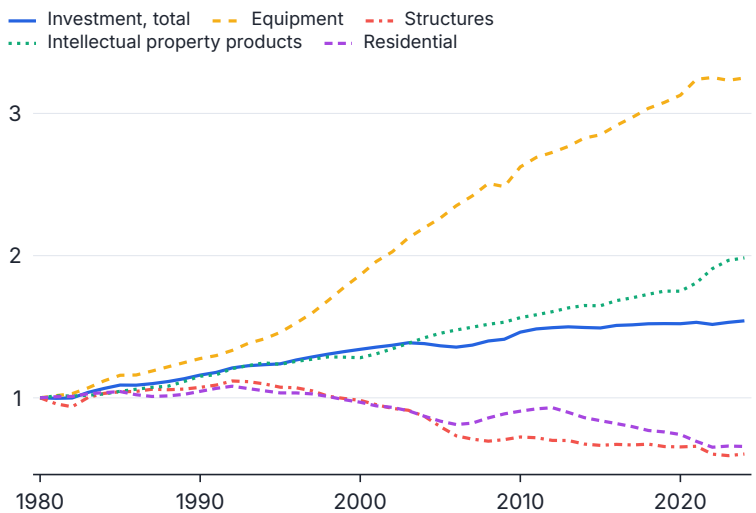
Figure 4. The relative price of consumption to investment, 1980–2024.



Notes: Ratio of the price index of personal consumption expenditures to the price index of gross private domestic investment, 1980 = 1. Source: BEA NIPA Table 1.1.4.

Figure 5 separates the investment price into its components, each relative to the consumption price. The rise of the relative price comes from equipment and IPP. Relative to consumption, equipment prices fall at 2.7% a year over 1980–2024 and IPP prices at 1.6% a year. Nonresidential structures and housing become dearer, at 1.1% and 1.0% a year. The aggregate index averages the two groups with their expenditure weights, which is why it moves less than either. The slowdown of the relative price after 2012 is also a mix of two things. The relative price of equipment falls at 1.5% a year, well below its 3.6% of 1997–2012. The relative price of housing rises at 2.9% a year, faster than before. The IPP price, by contrast, falls at a steady 1.4% to 1.8% a year throughout. This composition is behind the current-cost shares of Figure 1. The equipment share of the stock falls while its relative price falls. The IPP share rises despite a falling relative price, because IPP investment grew faster still.

Figure 5. The relative price of consumption to investment, by investment component, 1980–2024.



Notes: Price index of personal consumption expenditures over the price index of gross private domestic investment and of each of its components (nonresidential structures, equipment, intellectual property products, residential), 1980 = 1. Source: BEA NIPA Table 1.1.4.

3. The Model

The model in this section connects the two facts of Section 2, the decline of the labor share and the rise of the relative price of consumption to investment. The economy has two sectors. Sector C produces the consumption good and sector I produces the investment good, which is also the capital good. Each sector produces with a constant elasticity of substitution (CES) technology in capital and labor, with its own labor-augmenting technology level and its own elasticity of substitution. Markets are competitive. The structure is similar to that of the two-sector representation of the growth model with investment-specific technological change of Greenwood *et al.* (1997). In that representation both sectors produce with Cobb-Douglas technologies with the same factor shares, and the output of the investment sector is scaled by an investment-specific productivity factor. Here the two technologies are CES, differ in their distribution parameters, and each has its own labor-augmenting technology level. Preferences allow for an endogenous labor supply without wealth effects.

3.1. Setup

■ **Households.** A representative household has an intertemporal utility function of the Greenwood-Hercowitz-Huffman (GHH) form (Greenwood *et al.*, 1988):

$$U = \sum_{t=0}^{+\infty} \beta^t \frac{\left(c_t - \Gamma_t \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{1-\varphi}}{1-\varphi}, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $\varphi^{-1} > 0$ the elasticity of intertemporal substitution, $\eta^{-1} > 0$ the elasticity of labor supply, c_t consumption, ℓ_t hours worked, and Γ_t an exogenous scaling factor of the disutility of labor. Hours can be supplied to either sector, and the disutility of labor depends on their sum:

$$\ell_t \equiv \ell_{Ct} + \ell_{It}. \tag{2}$$

GHH preferences rule out wealth effects on labor supply, so hours respond only to the current wage, not to wealth, in line with the evidence that households with different wealth differ in consumption but not in hours (Attanasio *et al.*, 2012; Browning *et al.*, 1985).

The investment good is the numeraire. Let $q_t \equiv P_{Ct}/P_{It}$ denote the price of consumption relative to investment, the relative price documented in Section 2. The representative household owns the capital stock k_t, rents it to firms at the rental rate r_t, supplies hours at the wage w_t, and chooses consumption, hours, and next period's capital subject to

$$q_t c_t + k_{t+1} \leq (1 + r_t - \delta)k_t + w_t \ell_t, \tag{3}$$

where $\delta \in (0,1)$ is the depreciation rate. Firms earn no profits under constant returns and competitive markets, so no profit income appears in Equation 3.

Because the disutility of labor depends only on total hours, the household supplies hours to the sector that pays more until wages are equal, $w_{Ct} = w_{It} = w_t$, and the sector supplies are perfectly elastic at the common wage. Combining the first-order conditions for hours and consumption and solving for hours gives the aggregate labor supply,

$$\ell_t = \left(\frac{w_t}{\Gamma_t q_t}\right)^{\frac{1}{\eta}}, \tag{4}$$

and the one for capital gives the Euler equation,

$$\left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{\varphi} = \beta_\Gamma \frac{q_t}{q_{t+1}} \frac{1 + r_{t+1} - \delta}{\Gamma_*}, \quad \tilde{c}_t \equiv \frac{c_t}{\Gamma_t} - \frac{\ell_t^{1+\eta}}{1+\eta}, \tag{5}$$

where $\Gamma_* \equiv \Gamma_{t+1}/\Gamma_t$ is the growth factor of the scaling factor, constant by assumption, and $\beta_\Gamma \equiv \beta \Gamma_*^{1-\varphi}$. The household's optimum also includes the transversality condition

$$\lim_{t \to +\infty} \beta^t (\Gamma_t \tilde{c}_t)^{-\varphi} \frac{k_{t+1}}{q_t} = 0, \tag{6}$$

where $(\Gamma_t \tilde{c}_t)^{-\varphi}/q_t$ is the marginal utility of a unit of the investment good. The labor supply in Equation 4 depends on the wage in units of consumption, w_t/q_t, relative to the trend Γ_t: the GHH form makes hours a function of that detrended wage alone.

■ **Firms.** A representative firm in sector $j \in \{C, I\}$ produces with

$$y_{jt} = A_j \left[\alpha_j k_{jt}^{\rho_j} + (1 - \alpha_j)(z_{jt}\ell_{jt})^{\rho_j}\right]^{\frac{1}{\rho_j}}, \quad \rho_j \equiv \frac{\varepsilon_j - 1}{\varepsilon_j} < 1, \tag{7}$$

where k_{jt} and ℓ_{jt} are the capital and hours used in the sector, z_{jt} is the labor-augmenting technology level, $\varepsilon_j > 0$ the elasticity of substitution between capital and labor[1], $\alpha_j \in (0,1)$ a distribution parameter, and $A_j > 0$ a scale parameter. Technical change is labor-augmenting in both sectors. The technology of the consumption sector grows at the constant rate g_{z_C}. That of the investment sector grows at the rate $g_{z_I t} \equiv z_{It+1}/z_{It} - 1$. This rate may vary along a transition. The technology gap ζ_t is the ratio of the technology level of the investment sector to that of the consumption sector. Its definition and law of motion are

$$\zeta_t \equiv \frac{z_{It}}{z_{Ct}}, \quad \frac{\zeta_{t+1}}{\zeta_t} = \frac{1 + g_{z_I t}}{1 + g_{z_C}}. \tag{8}$$

[1] As is well known, for $\varepsilon_j = 1$ the limit of Equation 7 is the Cobb-Douglas technology $A_j k_{jt}^{\alpha_j} (z_{jt}\ell_{jt})^{1-\alpha_j}$.

The growth rate of the gap is the difference $g_{z_I t} - g_{z_C}$ divided by $1 + g_{z_C}$. The gap widens while the technology of the investment sector grows faster and is constant once the two rates are equal.

Let the effective capital intensity of sector j and the intensive-form production function be

$$\tilde{k}_{jt} \equiv \frac{k_{jt}}{z_{jt}\ell_{jt}}, \quad F_j(\tilde{k}) \equiv \left[\alpha_j \tilde{k}^{\rho_j} + (1-\alpha_j)\right]^{\frac{1}{\rho_j}}, \tag{9}$$

so that output is

$$y_{jt} = A_j z_{jt} \ell_{jt} F_j(\tilde{k}_{jt}). \tag{10}$$

The firm in sector j takes prices as given and chooses capital and hours to maximize profit, $q_t y_{Ct} - w_t \ell_{Ct} - r_t k_{Ct}$ in sector C and $y_{It} - w_t \ell_{It} - r_t k_{It}$ in sector I. The first-order conditions equate each marginal revenue product to the factor price:

$$q_t A_C (1-\alpha_C) z_{Ct} F_C(\tilde{k}_{Ct})^{\frac{1}{\varepsilon_C}} = w_t, \quad A_I (1-\alpha_I) z_{It} F_I(\tilde{k}_{It})^{\frac{1}{\varepsilon_I}} = w_t, \tag{11}$$

$$q_t A_C \alpha_C \left(\frac{F_C(\tilde{k}_{Ct})}{\tilde{k}_{Ct}}\right)^{\frac{1}{\varepsilon_C}} = r_t, \quad A_I \alpha_I \left(\frac{F_I(\tilde{k}_{It})}{\tilde{k}_{It}}\right)^{\frac{1}{\varepsilon_I}} = r_t. \tag{12}$$

Dividing the capital condition by the labor condition in each sector gives the effective capital intensity as a function of relative factor prices,

$$\tilde{k}_{jt} = \left[\frac{\alpha_j}{1-\alpha_j} \frac{w_t}{r_t z_{jt}}\right]^{\varepsilon_j}, \tag{13}$$

whose elasticity with respect to the wage per efficiency unit relative to the rental rate is ε_j.

With $\varepsilon_j < 1$ the marginal product of capital, the left-hand side of Equation 12 divided by the price of the sector's good, is bounded above[2],

$$\lim_{\tilde{k} \to 0} A_j \alpha_j \left(\frac{F_j(\tilde{k})}{\tilde{k}}\right)^{\frac{1}{\varepsilon_j}} = A_j \alpha_j^{\frac{\varepsilon_j}{\varepsilon_j - 1}}. \tag{14}$$

With $\varepsilon_j > 1$ the same expression is a lower bound.

[2] This bound limits the rental rates that a balanced growth path can support (Section 3.3).

The distribution parameter α_j is the weight of capital in the technology. In the Cobb-Douglas case it is the exponent of capital, and so also the elasticity of output with respect to capital. This elasticity is the capital cost share, because at the firm's optimum, by Equation 11 and Equation 12, it equals the share of capital in the cost of the firm. With a CES technology the capital cost share moves with the effective capital intensity, and it equals α_j at one intensity only. That intensity is the base point of the normalization described next. The parameters of a CES function are not invariant to the units in which its inputs are measured. Rescaling the technology level z_{jt} by a constant λ and replacing $1-\alpha_j$ by $(1-\alpha_j)\lambda^{\rho_j}$ leaves Equation 7 unchanged. The parameter α_j and the level of z_{jt} are therefore identified only jointly, and a value of α_j carries no meaning until the units of z_{jt} are fixed. Nor are the parameters comparable across values of ε_j. Changing the elasticity at fixed (A_j, α_j) changes output, the marginal products, and the factor shares at every input combination. Two technologies with different elasticities cannot be compared unless they agree somewhere. Klump *et al.* (2012) resolve both problems by writing the function relative to a base point $(y_{j0}, k_{j0}, z_{j0}\ell_{j0})$ at which the parameters are defined,

$$y_{jt} = y_{j0}\left[\alpha_j\left(\frac{k_{jt}}{k_{j0}}\right)^{\rho_j} + (1-\alpha_j)\left(\frac{z_{jt}\ell_{jt}}{z_{j0}\ell_{j0}}\right)^{\rho_j}\right]^{\frac{1}{\rho_j}},$$

so that α_j is the capital cost share at the base point for every ε_j. Equation 7 is this function with the base point chosen where the firm uses one unit of capital per efficiency unit of labor, $k_{j0} = z_{j0}\ell_{j0}$, and with $y_{j0} = A_j k_{j0}$. The two denominators then become equal and factor out, and what remains is the standard form. The normalization is therefore a choice of base point, $\tilde{k} = 1$, and of the units of z_{jt} that put the base period there. The function itself is unchanged. At that point $F_j(1) = 1$, so output equals $A_j k_{jt}$. The scale A_j is the output-capital ratio of sector j in units of its own good, and $A_j\alpha_j$ is the marginal product of capital. The parameter α_j is the capital cost share at the base point, whatever the elasticity of substitution. Two technologies with different ε_j and the same (A_j, α_j) produce the same output, pay the same factor shares, and have the same marginal products at $\tilde{k} = 1$. They differ only in how fast those objects move away from it. The normalization is also what allows the parameters to be read from data. A capital cost share ψ_{j0} observed in a base period at an effective capital intensity $\tilde{k}_{j0}$ identifies α_j given ε_j,

$$\alpha_j = \frac{\psi_{j0}}{\psi_{j0} + (1-\psi_{j0})\tilde{k}_{j0}^{\rho_j}}, \tag{15}$$

and the choice of units $z_{j0} = k_{j0}/\ell_{j0}$, which sets $\tilde{k}_{j0} = 1$, gives $\alpha_j = \psi_{j0}$, the base-period capital cost share. With the same units, A_j is the base-period output-capital ratio. Both readings are used in the calibration. Away from the base point, with $\varepsilon_j < 1$, the capital cost share is the function

$$\psi_j(\tilde{k}) \equiv \frac{\partial y_{jt}}{\partial k_{jt}} \frac{k_{jt}}{y_{jt}} = \frac{\alpha_j \tilde{k}^{\rho_j}}{\alpha_j \tilde{k}^{\rho_j} + 1 - \alpha_j}, \tag{16}$$

and the labor cost share is its complement, $1 - \psi_j(\tilde{k})$. For $\varepsilon_j < 1$ the capital cost share falls with the effective capital intensity,

$$\frac{\mathrm{d}\psi_j}{\mathrm{d}\tilde{k}} = \frac{\alpha_j(1-\alpha_j)\rho_j \tilde{k}^{\rho_j - 1}}{\left(\alpha_j \tilde{k}^{\rho_j} + 1 - \alpha_j\right)^2} < 0, \quad \lim_{\tilde{k}\to 0} \psi_j(\tilde{k}) = 1, \quad \lim_{\tilde{k}\to +\infty} \psi_j(\tilde{k}) = 0 : \tag{17}$$

capital deepening lowers the share of the factor that becomes abundant. For $\varepsilon_j > 1$ the derivative is positive. For $\varepsilon_j = 1$, $\psi_j(\tilde{k}) = \alpha_j$ at every $\tilde{k}$, and the normalization is immaterial.

3.2. General Equilibrium

An equilibrium is a sequence of allocations $\{c_t, i_t, k_{t+1}, k_{jt}, \ell_{jt}, y_{jt}\}_{t=0}^{+\infty}$ and prices $\{w_t, r_t, q_t\}_{t=0}^{+\infty}$ such that, given $k_0 > 0$, z_{C0}, g_{z_C}, $\{\zeta_t\}_{t=0}^{+\infty}$, and the parameters $[\beta, \varphi, \eta, \delta, A_j, \alpha_j, \varepsilon_j]$, firms maximize profits, the household maximizes utility, and markets clear. There are four markets. The household buys the consumption good at the price q_t, and the market for this good clears when the output of sector C equals consumption,

$$y_{Ct} = c_t. \tag{18}$$

The market for the investment good clears when the output of sector I equals gross investment,

$$y_{It} = i_t \equiv k_{t+1} - (1-\delta)k_t, \tag{19}$$

so the capital stock evolves as $k_{t+1} = (1-\delta)k_t + y_{It}$. The household rents its whole capital stock to the firms of the two sectors at the common rental rate r_t, and the market for capital clears when the two sectors use that stock,

$$k_{Ct} + k_{It} = k_t. \tag{20}$$

The household supplies its hours to the two sectors at the common wage w_t, and the market for hours clears when the two sectors use those hours,

$$\ell_{Ct} + \ell_{It} = \ell_t. \tag{21}$$

The equilibrium can be reduced[3] to a system of four equations in four endogenous variables, once the sector intensities and the relative price are written as functions of them. Let the shares of capital and hours used in the investment sector and the aggregate effective capital intensity be

[3] The system is written for the scaling factor $\Gamma_t = A_C z_{Ct}$, the form that Section 3.3 gives it.

$$s_{kt} \equiv \frac{k_{It}}{k_t}, \quad s_{\ell t} \equiv \frac{\ell_{It}}{\ell_t}, \quad \tilde{k}_t \equiv \frac{k_t}{z_{It}\ell_t}. \tag{22}$$

The sector intensities are then

$$\tilde{k}_{It} = \frac{s_{kt}}{s_{\ell t}}\tilde{k}_t, \quad \tilde{k}_{Ct} = \frac{1 - s_{kt}}{1 - s_{\ell t}}\tilde{k}_t\zeta_t. \tag{23}$$

The intensity of the investment sector depends on the endogenous variables only. The intensity of the consumption sector also depends on ζ_t, because capital is measured in efficiency units of the investment sector while the consumption sector's technology is z_{Ct}. This asymmetry is where the technology gap enters the dynamics, and it is absent under Cobb-Douglas technology, where ζ_t is absorbed into a common trend. The equilibrium path is characterized by these four variables.

Proposition 1. *(Reduced Equilibrium System).* Given $k_0 > 0$ and the path $\{\zeta_t\}_{t=0}^{+\infty}$, an equilibrium is a path $\{s_{\ell t}, s_{kt}, \ell_t, \tilde{k}_{t+1}\}_{t=0}^{+\infty}$ that satisfies, in every period,

$$\ell_t = \left[(1 - \alpha_C)F_C\big(\tilde{k}_{Ct}\big)^{\frac{1}{\varepsilon_C}}\right]^{\frac{1}{\eta}}, \tag{24}$$

$$\zeta_t^{1-\frac{1}{\varepsilon_C}}\tilde{k}_t^{\frac{1}{\varepsilon_I}-\frac{1}{\varepsilon_C}}\left(\frac{s_{kt}}{s_{\ell t}}\right)^{\frac{1}{\varepsilon_I}}\left(\frac{1 - s_{\ell t}}{1 - s_{kt}}\right)^{\frac{1}{\varepsilon_C}} = \mathcal{A}, \quad \mathcal{A} \equiv \frac{\alpha_I}{1 - \alpha_I}\frac{1 - \alpha_C}{\alpha_C}, \tag{25}$$

$$\begin{aligned}\left[\frac{(1 - s_{\ell t+1})F_C\big(\tilde{k}_{Ct+1}\big) - \ell_{t+1}^{\eta}/(1+\eta)}{(1 - s_{\ell t})F_C\big(\tilde{k}_{Ct}\big) - \ell_t^{\eta}/(1+\eta)}\frac{\ell_{t+1}}{\ell_t}\right]^{\varphi} = \\ = \frac{q_t}{q_{t+1}}\beta_\Gamma\frac{1 - \delta + A_I\alpha_I\big(F_I\big(\tilde{k}_{It+1}\big)/\tilde{k}_{It+1}\big)^{\frac{1}{\varepsilon_I}}}{\Gamma_*},\end{aligned} \tag{26}$$

$$\tilde{k}_{t+1}\big(1 + g_{z_I t}\big)\frac{\ell_{t+1}}{\ell_t} = (1 - \delta)\tilde{k}_t + A_I s_{\ell t}F_I\big(\tilde{k}_{It}\big), \tag{27}$$

where the sector intensities $\tilde{k}_{It}$ and $\tilde{k}_{Ct}$ stand for the functions of $\big(s_{kt}, s_{\ell t}, \tilde{k}_t, \zeta_t\big)$ in Equation 23 and the relative price q_t for the function of the two intensities by

$$q_t = \frac{A_I\alpha_I}{A_C\alpha_C}\left(\frac{F_I\big(\tilde{k}_{It}\big)}{\tilde{k}_{It}}\right)^{\frac{1}{\varepsilon_I}}\left(\frac{\tilde{k}_{Ct}}{F_C\big(\tilde{k}_{Ct}\big)}\right)^{\frac{1}{\varepsilon_C}}. \tag{28}$$

The remaining allocations and prices follow from the definitions in Equation 22, the technologies in Equation 7, market clearing in Equation 18 to Equation 21, the labor supply in Equation 4, and the capital condition of sector I in Equation 12.

Proof. See Appendix A.1. □

Equation 24 is the labor condition of sector C combined with the labor supply: hours depend only on the effective capital intensity of the consumption sector. Equation 25 is the equalization of the marginal rates of technical substitution across sectors, since both face the same w_t and r_t. Its left-hand side is strictly decreasing in $s_{\ell t}$, from $+\infty$ as $s_{\ell t} \to 0$ to 0 as $s_{\ell t} \to 1$, so it has a unique solution $s_{\ell t} \in (0,1)$ for given $\left(s_{kt}, \tilde{k}_t, \zeta_t\right)$. Equation 26 is the Euler equation, with the rental rate from the capital condition of sector I and the relative price from the ratio of the two capital conditions, Equation 28. Equation 27 is capital accumulation in efficiency units, with the growth rate of z_{It} allowed to vary over time. The scale A_C cancels throughout, since it enters the wage through Γ_t and the output of sector C by the same factor. It is a choice of units of the consumption good. A_I remains in the rental rate, the relative price, and capital accumulation, because capital and the output of sector I are the same good.

With $\varepsilon_C \neq 1$ the technology gap ζ_t enters Equation 24, Equation 25, and Equation 26 through $\tilde{k}_{Ct}$, so the system is not autonomous while the gap changes. And Equation 28 ties the relative price to the two effective capital intensities. Along a path with $g_{z_I t} > g_{z_C}$, ζ_t and hence $\tilde{k}_{Ct}$ rise, and q_t rises with them: the consumption sector becomes capital-intensive in efficiency units and its good becomes dearer relative to investment. This is the mechanism through which the widening of the technology gap produces the rise of q in Section 2.

3.3. Balanced Growth Path

This section characterizes the balanced growth path of the model and the conditions under which it exists. The paper assumes that the economy converges to such a path and uses the transition toward it to read the labor share. Along the path the labor share is constant, at any elasticity of substitution, so a labor share that moves is a transition. The mechanism of Section 3.2 acts while the technology gap ζ_t widens, and leaves a level effect through the level of the gap at which the transition ends. The conditions for the path to exist restrict the parameters, in particular the technology growth rates of the two sectors, and the computation imposes the path at the terminal period, so its properties are part of the quantitative exercise.

A balanced growth path is an equilibrium path along which all quantities grow at constant rates. Along it the reduced variables $\left(s_{kt}, s_{\ell t}, \ell_t, \tilde{k}_t\right)$ are constant, so a BGP corresponds to a steady state of the reduced system. A BGP requires the growth rate of the technology of the investment sector to be constant, written without the time subscript as g_{z_I}. Here a BGP

has a constant technology gap, $g_{z_I} = g_{z_C}$.[4] The paper assumes that the gap stops widening, so that the economy converges to a balanced growth path. The results below are conditional on that assumption.

The scaling factor of the disutility of labor grows with the labor-augmenting technology of the consumption sector,

$$\Gamma_t \equiv A_C z_{Ct}, \tag{29}$$

so that $\Gamma_* = 1 + g_{z_C}$. With CES technologies the growth factor of consumption is not a power function of the two technology levels, and Equation 29 is the trend under which hours are stationary whenever a BGP exists, as the proposition below shows.

In what follows the BGP has $g_{z_I} = g_{z_C} \equiv g$, $\zeta_t = \zeta_*$, $\Gamma_* = 1 + g$, and $\beta_\Gamma = \beta(1+g)^{1-\varphi}$. The level ζ_* is a parameter of the BGP.

Proposition 2. *(Balanced Growth Path).* Let $g_{z_I} = g_{z_C} \equiv g$, $\zeta_t = \zeta_*$, and $\varepsilon_I \neq 1$. The BGP is recursive. The rental rate is

$$r_* = \frac{(1+g)^\varphi}{\beta} - (1-\delta), \tag{30}$$

and, with $\bar{r}_* \equiv r_*/A_I$, the effective capital intensity of the investment sector is

$$\tilde{k}_{I*} = \left[\frac{\left(\bar{r}_*/\alpha_I\right)^{\varepsilon_I - 1} - \alpha_I}{1-\alpha_I}\right]^{\frac{\varepsilon_I}{1-\varepsilon_I}}, \tag{31}$$

the share of capital in sector I

$$s_{k*} = \frac{g+\delta}{A_I}\left(\frac{\alpha_I}{\bar{r}_*}\right)^{\varepsilon_I}, \tag{32}$$

the effective capital intensity of the consumption sector

$$\tilde{k}_{C*} = \left[\zeta_* \frac{\tilde{k}_{I*}^{1/\varepsilon_I}}{\mathcal{A}}\right]^{\varepsilon_C}, \tag{33}$$

the share of hours in sector I

[4] With $\varepsilon_C \neq 1$ this is a restriction, the two-sector form of the theorem of Uzawa (1961). Appendix A.6 states it, together with the Cobb-Douglas case in which any pair of growth rates is consistent with a BGP. The theorem is also why technical change is labor-augmenting here: with exogenous technology and an elasticity of substitution different from one, it is the only form that a balanced growth path admits.

$$s_{\ell *} = \frac{\tilde{k}_{C*} s_{k*}}{\tilde{k}_{C*} s_{k*} + \tilde{k}_{I*} \zeta_* (1 - s_{k*})}, \tag{34}$$

the aggregate intensity

$$\tilde{k}_* = \tilde{k}_{I*} \frac{s_{\ell *}}{s_{k*}}, \tag{35}$$

and hours

$$\ell_* = \left[(1 - \alpha_C) F_C \big(\tilde{k}_{C*} \big)^{\frac{1}{\varepsilon_C}} \right]^{\frac{1}{\eta}}. \tag{36}$$

The BGP exists, with $s_{k*} \in (0, 1)$ and a positive argument of the utility function, if and only if

$$\left(\frac{\bar{r}_*}{\alpha_I} \right)^{\varepsilon_I - 1} > \alpha_I, \quad g + \delta < A_I \left(\frac{\bar{r}_*}{\alpha_I} \right)^{\varepsilon_I}, \quad (1 - s_{\ell *})(1 + \eta) > 1 - \psi_C \big(\tilde{k}_{C*} \big). \tag{37}$$

These conditions do not restrict the elasticity of substitution of the consumption sector: with $g_{z_I} = g_{z_C}$ the BGP exists for any ε_C.

Proof. See Appendix A.2. □

For $\varepsilon_I < 1$ the first condition in Equation 37 is the bound on the marginal product of capital in Equation 14, $r_* < A_I \alpha_I^{\varepsilon_I / (\varepsilon_I - 1)}$. The rental rate that preferences and growth require must be attainable by the investment technology. For $\varepsilon_I > 1$ it is a lower bound on the same object, and for $\varepsilon_I = 1$ the intensity is $\tilde{k}_{I*} = \big(\bar{r}_* / \alpha_I \big)^{1/(\alpha_I - 1)}$ and the condition drops. The second condition keeps the share of capital in sector I below one. The third keeps consumption net of the disutility of labor positive. It holds whenever $s_{\ell *} < \eta / (1 + \eta)$. The first three steps of the recursion depend only on preferences, growth, and the technology of sector I, that is, on $(\beta, \varphi, \delta, g, A_I, \alpha_I, \varepsilon_I)$. These are the rental rate in Equation 30, the intensity in Equation 31, and the share of capital in Equation 32. The technology gap ζ_* enters from the fourth step on. It enters directly in the intensity of sector C in Equation 33, with elasticity ε_C, and in the share of hours in Equation 34. Through these it enters the aggregate intensity and hours.

Corollary 1. *(Growth Rates).* Along the BGP, capital, output of both sectors, consumption, investment, and the wage grow at the rate g. Hours, the relative price q_t, the rental rate r_t, and the sector capital cost shares ψ_{jt} are constant. The gross return satisfies $1 + r_* - \delta = (1 + g)^{\varphi} / \beta$.

Proof. See Appendix A.3. □

Consumption grows at g, the growth rate of $\Gamma_t = A_C z_{Ct}$ in Equation 29, so c_t/Γ_t is constant along the BGP and hours are constant with it. If the scaling factor grew at any other rate, hours would not be constant along the BGP.

The corollary states what the BGP restriction removes. On the BGP the relative price is constant, so the rise of q documented in Section 2 cannot be a BGP phenomenon when $\varepsilon_C < 1$. It is a transition: a period during which g_{z_I} exceeds g_{z_C}, ζ_t rises, and q_t rises with it, followed by convergence to equal growth rates. The shape of the path in Section 2 is consistent with this reading. A relative price rising at a constant rate would require a gap growing at a constant rate forever, which the proposition rules out. The rise of q instead slows down over the period. A growth rate that falls toward zero is what convergence to a new BGP produces, one with a higher ζ_* and a higher q_*. The next subsection shows that the labor share is constant along the BGP as well, so sectoral technology differences move it only along the transition and through the level of the gap ζ_* at which the transition ends.

3.4. Labor Share Determinants

Let $\psi_{jt} \equiv \psi_j(\tilde{k}_{jt})$ denote the sector capital cost shares along the equilibrium path. From the firms' conditions and Equation 16, labor income in each sector is

$$w_t \ell_{Ct} = (1 - \psi_{Ct}) q_t y_{Ct}, \quad w_t \ell_{It} = (1 - \psi_{It}) y_{It}, \tag{38}$$

and capital income is

$$r_t k_{Ct} = \psi_{Ct} q_t y_{Ct}, \quad r_t k_{It} = \psi_{It} y_{It}. \tag{39}$$

Since both sectors pay the same rental rate, the two expressions in Equation 39 give the relative size of the sectors in value,

$$\frac{q_t y_{Ct}}{y_{It}} = \frac{\psi_{It}}{\psi_{Ct}} \frac{1 - s_{kt}}{s_{kt}}. \tag{40}$$

Let aggregate output in units of the numeraire be

$$y_t \equiv q_t y_{Ct} + y_{It}, \tag{41}$$

and the nominal share of sector I be

$$\omega_t \equiv \frac{y_{It}}{y_t} = \frac{\psi_{Ct} s_{kt}}{\psi_{It}(1 - s_{kt}) + \psi_{Ct} s_{kt}}. \tag{42}$$

Proposition 3. *(Labor Share).* The labor share of income, $m_{\ell t} \equiv w_t \ell_t / y_t$, is

$$m_{\ell t} = \frac{(1-\psi_{Ct})\psi_{It}(1-s_{kt}) + (1-\psi_{It})\psi_{Ct}s_{kt}}{\psi_{It}(1-s_{kt}) + \psi_{Ct}s_{kt}}, \tag{43}$$

or, as a weighted average of the sector labor shares,

$$m_{\ell t} = (1-\omega_t)m_{\ell Ct} + \omega_t m_{\ell It}, \tag{44}$$

with $m_{\ell Ct} \equiv w_t \ell_{Ct}/(q_t y_{Ct}) = 1-\psi_{Ct}$ and $m_{\ell It} \equiv w_t \ell_{It}/y_{It} = 1-\psi_{It}$. In terms of factor prices, the sector labor shares are

$$m_{\ell jt} = \frac{\left(1-\alpha_j\right)^{\varepsilon_j}\left(w_t/z_{jt}\right)^{1-\varepsilon_j}}{\alpha_j^{\varepsilon_j} r_t^{1-\varepsilon_j} + \left(1-\alpha_j\right)^{\varepsilon_j}\left(w_t/z_{jt}\right)^{1-\varepsilon_j}}. \tag{45}$$

Proof. See Appendix A.4. □

The aggregate labor share moves through two channels. The first is within sectors: each sector's labor share is its labor cost share $1-\psi_{jt}$, which varies with the sector's effective capital intensity when $\varepsilon_j \neq 1$ and is constant at $1-\alpha_j$ when $\varepsilon_j = 1$. The second is composition: the weight ω_t of the investment sector, which depends on the share of capital in sector I, s_{kt}, and on the two capital cost shares. At given cost shares, the numerator and the denominator of Equation 43 are linear in s_{kt} with the same slope, $\psi_{Ct} - \psi_{It}$, and their difference is $\psi_{Ct}\psi_{It}$, so

$$\frac{\partial m_{\ell t}}{\partial s_{kt}} = \frac{(\psi_{Ct} - \psi_{It})\psi_{Ct}\psi_{It}}{\left(\psi_{It}(1-s_{kt}) + \psi_{Ct}s_{kt}\right)^2}. \tag{46}$$

A shift of capital toward sector I lowers the labor share if and only if $\psi_{Ct} < \psi_{It}$, that is, when the consumption sector is the one with the higher labor cost share, and leaves it unchanged when the two cost shares are equal. With $\varepsilon_j \neq 1$ the cost shares move with the effective capital intensities, so the parameters alone do not fix the sign of the composition channel. Equal distribution parameters, $\alpha_C = \alpha_I$, equalize the cost shares only at the base point, where both intensities are one. Away from it the intensities differ whenever $\zeta_t \neq 1$ or $\varepsilon_C \neq \varepsilon_I$, by Equation 25, and the composition channel is open. The ordering of the cost shares, and with it the sign of the channel, can change along a transition.

The within-sector channel is where sectoral technology differences act, and Equation 45 shows how. In sector I the wage per efficiency unit w_t/z_{It} is stationary whenever $\tilde{k}_{It}$ is, so $m_{\ell It}$ moves only with $\tilde{k}_{It}$ along the transition. In sector C the wage per efficiency unit is $w_t/z_{Ct} = \zeta_t w_t/z_{It}$ and inherits the trend of ζ_t. With $\varepsilon_C < 1$ and $g_{z_I} > g_{z_C}$, the term $(w_t/z_{Ct})^{1-\varepsilon_C}$ rises over time for given $(s_{kt}, s_{\ell t}, \tilde{k}_t)$, and with it the labor share of the consumption sector. Faster technology growth in the investment sector raises the wage faster than the technology of the consumption sector, so the cost of labor per efficiency unit

rises there, and with substitution below one the share of the factor whose relative cost rises goes up. This is the sense in which sectoral technology differences are a force that raises the labor share, against the fall documented in Section 2, as long as the gap widens. On the BGP of Proposition 2 both intensities and both shares are constant, and so is $m_{\ell *}$:

$$m_{\ell *} = \frac{(1-\psi_{C*})\psi_{I*}(1-s_{k*}) + (1-\psi_{I*})\psi_{C*}s_{k*}}{\psi_{I*}(1-s_{k*}) + \psi_{C*}s_{k*}}, \tag{47}$$

where ψ_{I*} and s_{k*} depend only on preferences, growth, and the technology of sector I, by Equation 31 and Equation 32. The share ψ_{C*} depends additionally on ζ_*, α_C, and ε_C, by Equation 33. The level ζ_* affects the BGP labor share only through the CES channel in the consumption sector.

3.5. The Cobb-Douglas Case

The sign of the within-sector channel depends on which side of one the elasticity of substitution lies. The reading of the decline through cheaper capital, as in Karabarbounis & Neiman (2014), requires an elasticity above one: only then does a lower rental rate lower the share of labor. Values above one are hard to find in firm and industry data, which put the elasticity below one, as Section 1 reviews. With the lower values the same fall in the relative price raises the labor share rather than lowering it, which is the countervailing force of Section 3.4. Cobb-Douglas technology is the boundary between the two readings: at an elasticity of one the within-sector channel is closed in both directions, and what remains of the mechanism of this paper is the composition channel.

Corollary 2. *(Cobb-Douglas Technology).* Let $\varepsilon_C = \varepsilon_I = 1$. Then the technology of sector j is $y_{jt} = A_j k_{jt}^{\alpha_j}\left(z_{jt}\ell_{jt}\right)^{1-\alpha_j}$. The parameter α_j is the elasticity of output with respect to capital, which no longer varies with the effective capital intensity. It coincides with the capital cost share in every period, not only at the base point: $\psi_{jt} = \alpha_j$ for every t. The sector labor shares are constant at $1-\alpha_j$, and the aggregate labor share is

$$m_{\ell t} = \frac{\alpha_I(1-\alpha_C)(1-s_{kt}) + \alpha_C(1-\alpha_I)s_{kt}}{\alpha_I(1-s_{kt}) + \alpha_C s_{kt}}. \tag{48}$$

Proof. See Appendix A.5. □

The corollary isolates what the elasticity of substitution adds. Under Cobb-Douglas technology the within-sector channel is closed: sectoral technology differences change the labor share only through composition, as s_{kt} moves along the transition and settles at s_{k*}. The direction of that change is fixed by the parameters, since Equation 46 with $\psi_{jt} = \alpha_j$ is

$$\frac{\partial m_{\ell t}}{\partial s_{kt}} = \frac{(\alpha_C - \alpha_I)\alpha_C\alpha_I}{\left(\alpha_I(1-s_{kt}) + \alpha_C s_{kt}\right)^2} : \tag{49}$$

a rising s_{kt} lowers the labor share if and only if $\alpha_C < \alpha_I$. With $\alpha_C = \alpha_I$ the composition channel is closed as well, and the labor share is constant at $1 - \alpha$ whatever the technology gap. This is the condition on the cost shares of Section 3.4 with the cost shares fixed at the parameters, so that the ordering holds in every period. With $\varepsilon_j < 1$, by contrast, a technology gap moves the sector labor shares themselves while it widens. How much of the facts of Section 2 this transition accounts for is a quantitative question.

4. Results

The model is simulated at an annual frequency, with period 0 in 1980, and the relative price of Section 2.2 is the only series it is fitted to. The fit identifies the path of the technology gap between the sectors. The labor share and the other variables of the economy follow from it and are compared with the data afterwards. This section first sets the parameters and then reads the simulated paths against the facts of Section 2.

4.1. Calibration

The parameters fall in three groups: preferences, depreciation, and the elasticity of substitution, set from external estimates and aggregate data; the technology parameters and the technology growth rates, read from the data through the model; and the initial conditions. Table 2 collects the values.

The discount factor is $\beta = 0.96$, the value of Prescott (1986). The elasticity of intertemporal substitution is $\varphi^{-1} = 0.35$. It is the midpoint of the range that Vissing-Jørgensen (2002) estimates for stockholders, 0.3 to 0.4. The elasticity of labor supply is $\eta^{-1} = 0.75$. With GHH preferences this is the Frisch elasticity of aggregate hours. The value is the one that Chetty *et al.* (2011) recommend for models with a representative household, the sum of 0.5 on the intensive margin and 0.25 on the extensive margin. It lies between the 0.25 that the meta-analysis of Elminejad *et al.* (2023) finds most consistent with the microeconomic estimates and the range of 1 to 2 that Keane & Rogerson (2012) report at the macroeconomic level. The depreciation rate is the 1980–2024 mean of the depreciation rate of all private fixed assets in panel (B) of Figure 1, $\delta = 5.5\%$ a year. The model holds it constant while the data show it rising, from 4.7% to 5.7%. The gross labor share of the model inherits this simplification.

The elasticity of substitution is the same in both sectors, $\varepsilon_C = \varepsilon_I = \varepsilon$, with a baseline of 0.5. The value is the center of the range of Chirinko (2008) and the lower end of the aggregate estimates of Oberfield & Raval (2021). Four other values are simulated: 0.3, 0.7, 1 and 1.25. The first is the plant-level range of Oberfield & Raval (2021), the second the upper end of their aggregate range, the third the Cobb-Douglas case of Section 3.5, and the last the cross-country estimate of Karabarbounis & Neiman (2014), the value that the reading of the decline through cheaper capital requires. The value of one is also within the estimates: the cross-country point estimates of Glover & Short (2020) are near one and statistically

indistinguishable from it. The profile of the growth rate of the gap, described below, is refitted at each value, so that every case reproduces the relative price and differs from the others only in what the technology implies for the rest of the economy.

The technology parameters are read from data with the normalization of Section 3.1. The two sectors are mapped to industries. The investment sector is construction, durable goods manufacturing, and information. These industries produce structures, equipment, and the software and originals that make up IPP. The consumption sector is every other private industry. The industry accounts of the BEA that give value added and compensation by industry on this classification start in 1997. In that year the labor share of the consumption sector is 51.1% and that of the investment sector 58.6%. Under competitive markets the capital cost share of a sector is one minus its labor share, and Equation 15 with the base point $\tilde{k}_{j0} = 1$ gives α_j as that share: $\alpha_C = 0.489$ and $\alpha_I = 0.414$. The ordering and the distance between the sectors are those of Basu *et al.* (2013). They report capital shares of 0.36 for nondurables and services consumption and 0.30 for equipment investment, averaged over 1961–2004. The consumption sector is the more capital-intensive in both, by 0.07 here against 0.06 there. Valentinyi & Herrendorf (2008) find the same ordering through the input-output tables, 0.35 for consumption against 0.28 for investment. The levels here are higher because the two measures differ. Theirs are shares of vertically integrated final-use sectors, computed through the input-output structure. The ones here are compensation over the gross value added of the industries that produce each good, which leaves depreciation and proprietors' income on the capital side. The base point is placed in 1980, the first period of the simulation, and the 1997 cost shares stand in for the 1980 ones, which the industry accounts do not provide. The HP trend of the private-sector labor share is 52.9% in 1980 (53.7% raw) and 52.5% in 1997, so the use of the 1997 shares in place of the 1980 ones moves the aggregate by less than 0.4 pp. The 1997 values are used because they are the earliest available, not as evidence that the 1980 shares were the same.

The mapping of the sectors to industries is an approximation. The relative price is defined by final use, as the ratio of the deflators of personal consumption expenditures and of gross private domestic investment, while the sectors are defined by industry. Durable goods manufacturing also produces consumer durables, and part of the output of information is consumed. Industries of the consumption sector produce investment, among it research and development, own-account software, and the distribution margins on equipment. The capital side of the consumption sector also holds the income of owner-occupied housing, proprietors' income, and taxes on production. These raise α_C, and with it the difference between the two cost shares that signs the composition channel. The aggregate investment deflator also moves with the composition of investment. The rise of q comes from equipment and IPP, while the prices of structures and housing rise relative to consumption (Figure 5). The fit attributes a shift in that composition to the gap. The sector inputs are not rebuilt on a final-use basis through the input-output tables, as Valentinyi & Herrendorf (2008) do, and the gap is not fitted to the price of equipment and IPP alone. The

output scale of the investment sector is set so that the capital-output ratio of the model in 1980 equals the value in the data, 3.11 in investment units, at the equilibrium of period 0 described below. This gives $A_I = 0.367$, the same to three decimals at every elasticity. The scale of the consumption sector is a choice of units, $A_C = 1$.

The technology growth rates are the parameters that remain. The common rate g is not identified by the relative price, which depends on the difference between the sector rates and not on their level. It is set from the BGP. By Corollary 1 consumption per household grows at g along it. The trend growth of consumption per household in consumption units over 1980–2024 is 1.70% a year, the slope of an OLS regression of its logarithm on a linear time trend. The growth rate of the gap decays exponentially,

$$g_{z_I t} - g_{z_C} = \gamma_0 e^{-t/\tau}, \tag{50}$$

so that $g_{z_I t}$ converges to g and the economy converges to the BGP of Proposition 2 as $t \to +\infty$. This is the assumption, stated in Section 3.3, that the gap stops widening. The two parameters of the profile, the initial growth rate of the gap γ_0 and the time constant τ, are chosen by least squares on the logarithm of the relative price, indexed to 1980, over 1980–2024. The transition is computed over $T = 200$ periods, with the BGP imposed at the terminal period, and the first 45 periods are compared with the data. For the BGP to hold exactly at T, the computation shifts the profile of Equation 50 to $\gamma_0\left(e^{-t/\tau} - e^{-T/\tau}\right)/\left(1 - e^{-T/\tau}\right)$, which is zero at T and differs from the exponential by 0.3% of γ_0 at the fitted τ. The fitted parameters are those of the shifted profile.

The initial conditions follow from the same base point. Both sectors are placed at their base point in 1980, $\tilde{k}_{I0} = \tilde{k}_{C0} = 1$. With common factor prices, Equation 13 for the two sectors then fixes the initial technology gap at the ratio of the capital-to-labor cost ratios of the two sectors, $\zeta_0 = \mathcal{A} = 0.74$. The 1980 allocation of capital and hours across sectors is not a target. By the industry classification above, the investment sector holds 11.7% of the private capital stock in 1980 and 23.7% of hours in 1990, the first year of the hours series. Under common factor prices the ratio of capital per hour between the two sectors equals the ratio of their capital-to-labor cost ratios, for any technology. The 1980 stocks and the 1990 hours put the ratio of capital per hour at 0.43. The 1997 cost shares put the ratio of cost ratios at 0.74. No base point matches both, and the cost shares are kept because they define the technology. The economy starts from its 1980 state and not from a balanced growth path. The initial capital stock is the level that puts the consumption sector at its base point in the model's 1980, $\tilde{k}_{C0} = 1$. With $\zeta_0 = \mathcal{A}$ this puts the investment sector at $\tilde{k}_{I0} = 1$ as well. The aggregate effective capital intensity is then $\tilde{k}_0 = 1.27$ at the baseline elasticity, and the capital stock delivers it at the equilibrium hours of that period. Households and firms have perfect foresight, so the period-0 equilibrium depends on the whole path of the gap. This condition and the one on the capital-output ratio are therefore solved jointly with the fit of the growth rate of the gap. The level of z_{C0} is set to one, which with $A_C = 1$ and $\tilde{k}_{C0} =$

1 fixes the units of the consumption good and of capital. The initial technology level of the investment sector follows, $z_{I0} = \zeta_0 z_{C0} = 0.74$, and $k_0 = \tilde{k}_0 z_{I0} \ell_0$. In the model's 1980 the capital-output ratio is 3.11, the value in the data, by the choice of A_I. The investment sector holds 18.6% of capital and 23.5% of hours, against 11.7% and 23.7% in the data. The sector labor shares are the 1997 cost shares. The state is close to the balanced growth path consistent with ζ_0, given by Proposition 2 with $\zeta_* = \zeta_0$, but not on it. That path has $\tilde{k}_* = 1.29$ and a capital-output ratio of 3.15. The economy therefore starts 1.8% below the balanced-growth intensity, with a rental rate of 15.2% against 14.8% along that path. With a constant gap the economy converges to that path and the labor share moves along the way. The constant-gap path, the same economy with $g_{z_I t} = g$ from 1980 on, is therefore reported with the fitted path, and the difference between the two is the effect of the widening gap.

Table 2. Calibration.

Parameter	Value	Source or target
Preferences and depreciation		
Discount factor, β	0.96	Prescott (1986)
Elasticity of intertemporal substitution, $1/\varphi$	0.35	Midpoint of the stockholder estimates of Vissing-Jørgensen (2002), 0.3 to 0.4
Elasticity of labor supply, $1/\eta$	0.75	Frisch elasticity of aggregate hours, Chetty *et al.* (2011)
Depreciation rate, δ	5.5%	Mean depreciation rate of private fixed assets, 1980–2024
Technology		
Distribution parameter, sector C, α_C	0.489	Capital cost share of sector C in 1997
Distribution parameter, sector I, α_I	0.414	Capital cost share of sector I in 1997
Output scale, sector I, A_I	0.367	Capital-output ratio of 1980, 3.11, at $\tilde{k}_{C0} = 1$
Output scale, sector C, A_C	1	Units of the consumption good
Elasticity of substitution, ε (baseline)	0.5	Chirinko (2008), Oberfield & Raval (2021) sensitivity: 0.3, 0.7, 1 and 1.25
Growth and initial conditions		
Common technology growth rate, g	1.70%	Trend growth of consumption per household, 1980–2024
Initial growth rate of the gap, γ_0	3.36%	Least squares on the relative price, 1980–2024
Time constant of its growth rate, τ (years)	34.0	Least squares on the relative price, 1980–2024
Initial effective capital intensity, $\tilde{k}_0$	1.27	$\tilde{k}_{C0} = 1$ at the 1980 equilibrium. Data, not targeted: $s_k = 11.7\%$ (1980), $s_\ell = 23.7\%$ (1990)
Initial technology gap, ζ_0	0.74	Cost-share ratio $\mathcal{A}$ of Equation 25 at $\tilde{k}_{j0} = 1$

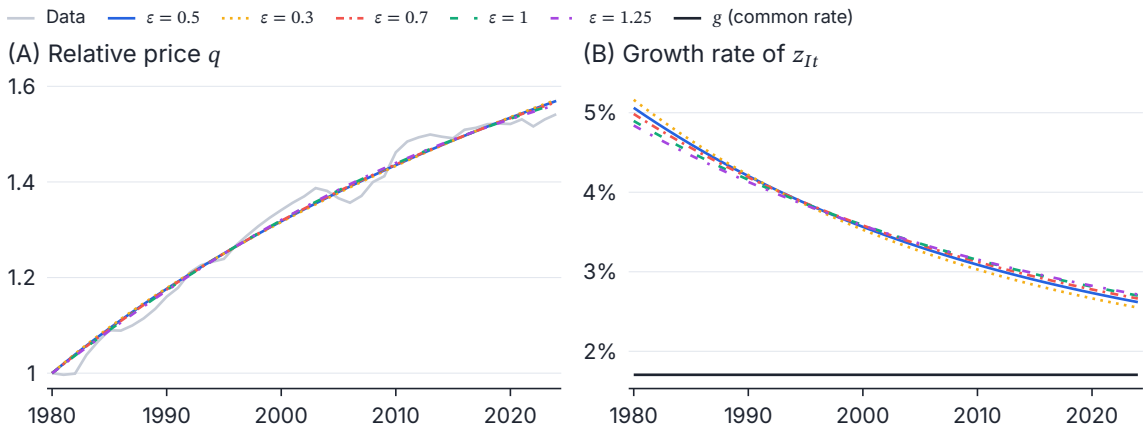
Notes: Sector I is construction (NAICS 23), durable goods manufacturing (33DG), and information (51). Sector C is the other private industries. Cost shares: compensation over value added by industry (BEA GDP by Industry), 1997 being the first year available. Capital shares by sector: current-cost net stock of private fixed assets by industry (BEA Fixed Assets Table 3.1ESI), 1980. Hours by sector: BLS Current Employment Statistics, available from 1990. The two allocation shares are reported for comparison with the model's 1980 allocation, which is not a target. Capital-output ratio: private fixed assets over consumption plus private investment, per household, in investment units. Consumption per household: personal consumption expenditures deflated by its own price index and divided by the number of households. Trend growth from an OLS regression of its logarithm on time. The initial growth rate of the gap and the time constant are the baseline values. The other cases are in Table 3 and Figure 6.

4.2. Simulation Analysis

This section describes the results of the model at five values of the elasticity of substitution. Four are in the range of the estimates in the literature. The fifth is the estimate above one with which Karabarbounis & Neiman (2014) explain about half of the decline of the labor share. Figure 6 shows the relative price in the model and in the data at the five elasticities, each with its own fitted profile of the growth rate of the gap. The baseline is $\varepsilon = 0.5$ in both sectors. The other four values are the sensitivity cases of Section 4.1. There the growth rate of the gap is 3.4% a year in 1980, with the investment sector's technology growing at 5.1% against 1.7% for the consumption sector's, and it halves every 23 years. The root mean squared error of the log relative price is 1.7 log points. By phase, the model matches the 25 log points of 1980–1997 with 25, falls short of the 15 of 1997–2012 with 13, and overshoots the 3 of 2012–2024 with 7. A single exponential cannot slow down as fast as the data did after 2012, and the fitted path keeps rising. The relative price on the terminal BGP is 1.91 times its 1980 level, against 1.54 in 2024. The elasticity changes little in this part. The initial growth rate of the gap ranges from 3.1% to 3.5% a year across the five cases and the error from 1.5 to 1.7 log points. The relative price in Equation 28 responds to the technology gap in every case, and the profile adjusts to deliver the same path of q.

Figure 7 shows the variables that are not used in the calibration, with the data of Section 2 and the per-household series of the national accounts. Panel (A) is the labor share. In the baseline the share starts at 52.7%, close to the HP trend of the private-sector gross measure in 1980, 52.9% (53.7% raw). It rises by 0.2 pp to 2000, by 1.9 pp over 2000–2012, and by 2.1 pp over 2012–2024. Over the whole period it rises by 4.2 pp while the data fall by 4.0 pp. The two paths part where the decline in the data is concentrated: over 2000–2012 the model adds 1.9 pp and the measures lose between 2.0 and 4.4 pp. Panels (B) and (C) show where the model's rise comes from. The labor share of the investment sector falls over the first two decades and then recovers part of the fall. While the gap widens fast, z_{It}

Figure 6. The relative price and the technology growth rates, by elasticity of substitution.

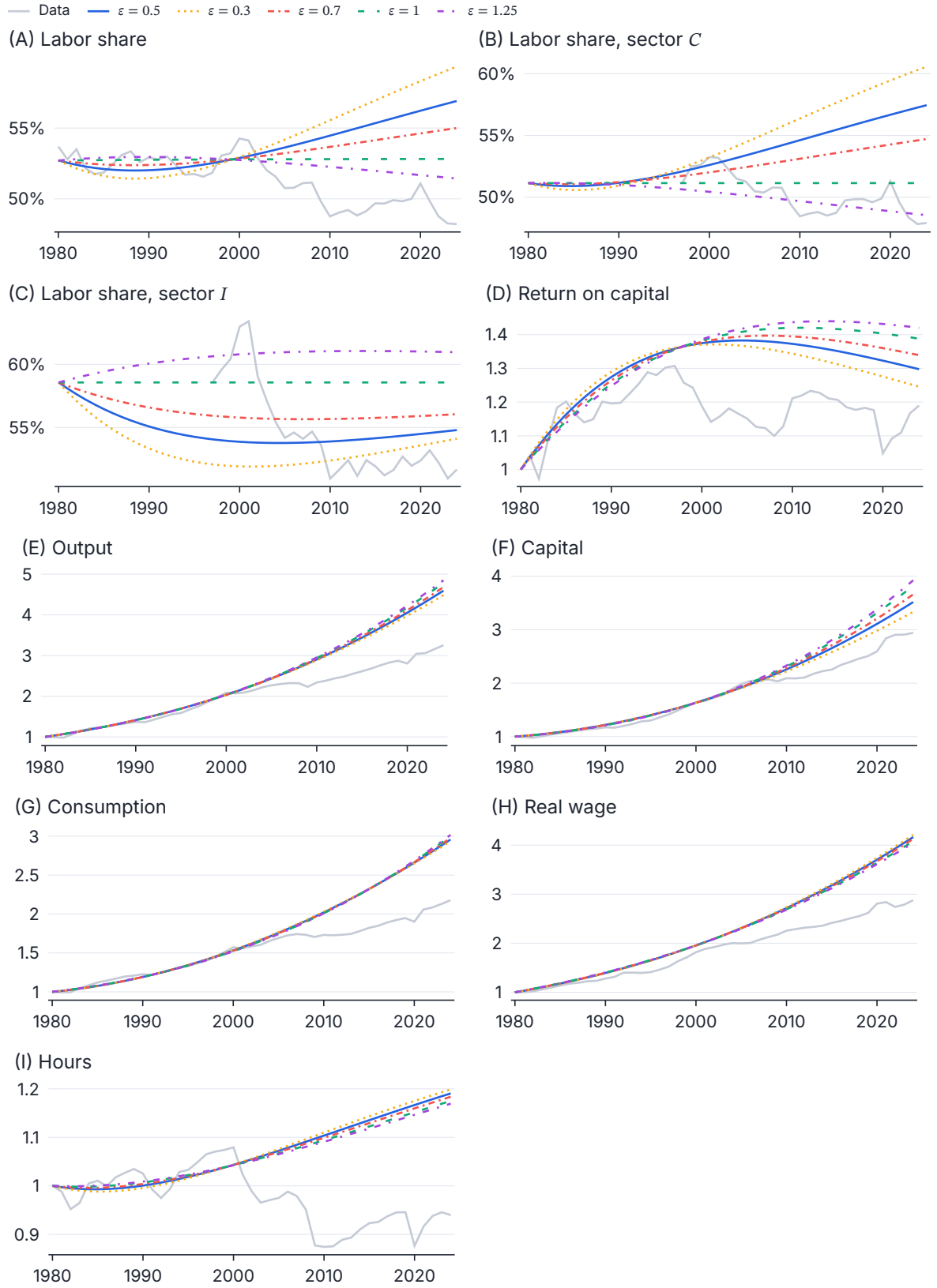


Notes: Annual, 1980–2024. (A) The relative price in the data (Figure 4) and in the model, 1980 = 1, one path per elasticity, each with its own profile of the growth rate of the gap fitted to the data. (B) The growth rate of the investment sector's technology under each fitted profile and the common rate g, in black, to which it converges.

grows faster than the capital stock, and the effective capital intensity of that sector falls below its base-point value of one. With $\varepsilon < 1$ a lower intensity means a higher capital cost share, by Equation 17. The labor share of the consumption sector falls slightly in the first years, while the fall of the aggregate intensity outweighs the rise of ζ_t in $\tilde{k}_{Ct}$. It then rises, by 5.4 pp over 1997–2024. This is the within-sector channel of Section 3.4: the widening gap raises the wage per efficiency unit of that sector, w_t/z_{Ct}, and with substitution below one the share of labor rises with its cost. In the data both sector shares fall over the same years, the consumption sector's by 4.1 pp and the investment sector's by 9.6 pp. The composition channel of Equation 44 is small and changes sign along the path. The share of capital in the investment sector rises from 18.6% to 21.4% in the model, as the faster-growing sector absorbs capital, while in the data it falls from 11.7% to 10.2%. The model's 1980 value is the one of the base point, above the measured share, as noted in Section 4.1. The consumption sector has the higher capital cost share at the start, $\psi_{C0} = 48.9\%$ against $\psi_{I0} = 41.4\%$, so by Equation 46 the shift raises the aggregate. The within-sector movements narrow that difference and reverse it late in the sample, as panels (B) and (C) show, after which the same shift lowers the aggregate.

Panels (D) to (I) and Table 3 cover the remaining variables. Output, capital, consumption, and the wage all grow faster in the model than in the data over the whole period. The wage runs ahead from the start, output and consumption from the early 2000s, and capital from the late 2000s. Before those years the data grow faster. Consumption per household grows at 2.5% a year in the baseline, above the 1.70% trend from which g was set, because the economy is not on its BGP, where the two would be equal. By Equation 10 consumption is $A_C z_{Ct} \ell_{Ct} F_C(\tilde{k}_{Ct})$, and only z_{Ct} grows at g. Along the transition ℓ_{Ct} and $F_C(\tilde{k}_{Ct})$ grow as well. Hours in the consumption sector add 0.5 pp a year, for the reason given with panel (I) below. The effective capital intensity of the sector adds 0.3 pp a year. Capital accumulates at the pace of the investment technology, faster than z_{Ct}, so $\tilde{k}_{Ct}$ rises from 1.00 to 1.29 and the consumption sector deepens its capital. Both terms vanish on the BGP, and they last as long as the gap widens. The fitted profile makes the widening slow to stop. The distance of a model path from the data is the root mean squared difference between the two changes since 1980 over the period. The wage shows the largest distance, 21 log points, and capital the smallest, 7. The return on capital of the model is the rental rate net of depreciation. It rises from 9.7% in 1980 to 12.6% in 2024, toward the level that the Euler equation requires while consumption grows faster than g. The data series divides all nonlabor income by the capital stock and runs between 13% and 14%. Under constant returns and competitive markets the two coincide, so panel (D) compares their changes as indices. The gap between their levels is the nonlabor income beyond the rental rate, which this model has none of. Over the period the return rises to 1.30 times its 1980 level in the model and to 1.01 times in the data. Hours per household, in panel (I), rise by 19% in the model over 1980–2024 and fall by 8% in the data. With GHH preferences hours follow the wage in units of consumption, w_t/q_t, relative to the trend $\Gamma_t = A_C z_{Ct}$ in Equation 4, and the widening gap raises that wage

Figure 7. The untargeted variables: model paths by elasticity of substitution against the data.



Notes: Annual, 1980–2024. Model paths per elasticity, each with its own profile of the growth rate of the gap. The data in gray. (A) Private-sector gross labor share of Figure 2, in %. (B), (C) Compensation over value added of the industries of each sector, available from 1997, in %. (D) Net of depreciation, 1980 = 1. Model: rental rate minus depreciation. Data: private value added net of compensation over private fixed assets, minus depreciation, which includes all nonlabor income. (E) to (G) Per household (Census, Table HH-1), 1980 = 1: output is personal consumption expenditures plus private fixed investment, and capital the current-cost net stock of private fixed assets (Fixed Assets Table 1.1), both deflated by the price index of gross private domestic investment. Consumption is deflated by its own price index. (H) Compensation of employees of corporate and noncorporate business (NIPA Table 1.13) over BLS hours worked, deflated by the investment price index, 1980 = 1. (I) BLS hours worked over the number of households, 1980 = 1.

faster than z_{Ct} for as long as it widens. Hours are thus one more variable that technology differences move against the data.

Table 3. Changes over 1980–2024, distance of the model paths from the data, and the fitted profile of the growth rate of the gap, by elasticity of substitution.

Variable	Unit	Data	$\varepsilon = 0.3$	$\varepsilon = 0.5$	$\varepsilon = 0.7$	$\varepsilon = 1$	$\varepsilon = 1.25$
Change 1980–2024							
Labor share	pp	−4.0	6.6	4.2	2.3	0.1	−1.3
Capital per household	log points	110.3	120.4	125.8	129.8	134.1	136.7
Output per household	log points	110.9	150.1	152.5	154.4	156.5	157.9
Consumption per household	log points	71.8	107.6	108.4	109.1	109.9	110.4
Real wage	log points	103.2	143.8	142.7	141.8	140.6	139.8
Return on capital	log points	0.6	22.0	26.1	29.2	32.8	35.1
Labor share, constant-gap path	pp		0.6	0.4	0.2	0.0	−0.2
Labor share, effect of the widening gap	pp		6.0	3.8	2.1	0.1	−1.1
Distance of the changes since 1980 from the data, 1980–2024 (RMSE)							
Labor share	pp		5.0	3.9	3.1	2.2	1.6
Capital per household	log points		4.8	6.9	8.4	10.1	11.1
Output per household	log points		20.8	21.7	22.4	23.2	23.7
Consumption per household	log points		17.6	17.6	17.7	17.8	17.9
Real wage	log points		21.9	21.4	21.1	20.6	20.4
Return on capital	log points		22.2	23.4	24.4	25.6	26.4
Fitted profile of the growth rate of the gap							
Initial growth rate of the gap, γ_0	% a year		3.46	3.36	3.28	3.19	3.14
Time constant, τ	years		31.3	34.0	36.1	38.2	39.3
Half-life of its growth rate	years		22	23	25	26	27
Error of the log relative price (RMSE)	log points		1.7	1.7	1.6	1.5	1.5

Notes: Changes: 2024 minus 1980 for ratios (pp) and log change of the per-household index for quantities (log points). For the data, the changes and the distances are computed on the HP trend of each series ($\lambda = 400$, fitted on the full sample), so the labor share change is the trend change of Table 1, −4.0 pp, where the raw change is −5.5 pp. Constant-gap path: the same economy with $g_{z_I t} = g$ from 1980 on. Effect of the widening gap: the fitted path minus the constant-gap path. Distance: root mean squared difference between the change since 1980 of the model path and that of the data over 1980–2024, in the same units. Fitted profile of the growth rate of the gap: the two parameters of Equation 50 chosen by least squares on the logarithm of the relative price at each elasticity, the half-life of its growth rate in years, and the root mean squared error of the log relative price. Data and model counterparts as in Figure 7.

The elasticity of substitution sets the size of the within-sector channel. The labor share rises by 6.6 pp at $\varepsilon = 0.3$, 4.2 pp at 0.5, 2.3 pp at 0.7, and 0.1 pp at 1, and falls by 1.3 pp at 1.25. At $\varepsilon = 1$ both sector shares are constant, by Corollary 2, and the 0.1 pp is the composition channel alone, which is complete by 2000: the share of capital in the investment sector settles early and the aggregate share with it. Above one the within-sector channel reverses sign (Section 3.5). The cost of labor per efficiency unit rises in the consumption sector as

before, and with substitution above one the share of labor in that sector falls. At $\varepsilon = 1.25$ it falls by 2.1 pp over 1997–2024, where it rises in the other cases, and the aggregate share falls from 2000 on. As ε falls below one, the within-sector channel opens and grows. At $\varepsilon = 0.3$ the share rises by 0.2 pp to 2000 and by 6.4 pp after, against 0.2 and 4.0 pp in the baseline. The convergence from the 1980 state is a small part of these changes. The constant-gap path adds 0.6 pp at $\varepsilon = 0.3$, 0.4 pp at 0.5, and 0.2 pp at 0.7. At $\varepsilon = 1$ it adds 0.0 pp, because there the labor share does not depend on the capital intensity. At $\varepsilon = 1.25$ it adds -0.2 pp. The effect of the widening gap is the rest: 6.0, 3.8, 2.1, 0.1, and -1.1 pp. The long-run effect of the gap is also ordered by the elasticity. The BGP labor share depends on the technology gap through Equation 33. In the baseline it moves from 53.4% at ζ_0 to 64.1% at the terminal level. It moves to 67.8% at $\varepsilon = 0.3$ and to 59.7% at 0.7, does not move at $\varepsilon = 1$, and falls to 47.3% at 1.25. The transition of 1980–2024 covers a part of the way. The rest is a level effect that outlasts the widening: a gap that widens for a time stays wider permanently. With $\varepsilon_C < 1$ the BGP labor share rises with it, and with $\varepsilon_C > 1$ it falls. No case reproduces the decline of 4.0 pp in the data. The share falls only at $\varepsilon = 1.25$, and there by 1.3 pp. The distance of the labor share path from the data is smallest at $\varepsilon = 1.25$, 1.6 pp, then 2.2 pp under Cobb-Douglas technology, where the path is flat, and it grows as the elasticity falls, to 5.0 pp at $\varepsilon = 0.3$. The smaller distances do not make the cases at and above one a better account of the data: the flat path is nearer to the decline of 4.0 pp only because it moves less, and the falling path covers a third of it.

The fall of 1.3 pp at $\varepsilon = 1.25$ is smaller than the effect that Karabarbounis & Neiman (2014) attribute to the relative price. Their model delivers about half of a decline of 5 pp in the global labor share from a fall of 25% in the relative price of investment. That computation compares steady states, across which the rental rate of capital falls in proportion to the price of investment. The same comparison in this model also gives a large effect. Between the BGP at ζ_0 and the terminal BGP, the relative price rises by 56 log points and the labor share falls by 5.3 pp, with the rental rate at r_* on both. Over 1980–2024 the relative price rises by 45 log points and the labor share falls by 1.3 pp. The difference comes from the rental rate. While the gap widens, the effective capital intensity of the investment sector falls and the rental rate in units of the investment good rises above its BGP value, from 15.2% in 1980 to 19.3% in 2024, a rise of 24 log points. The rental rate in units of consumption, r_t/q_t, therefore falls by 21 log points, about half of the rise of the relative price. By Equation 12 the capital cost share of sector C responds to that ratio, and the labor share of the sector falls by 2.6 pp, against 7.3 pp between the two BGPs. In sector I the higher rental rate lowers the capital cost share, and the labor share of that sector rises by 2.4 pp. Hours also move to sector I, the sector with the higher labor share: its share of hours rises from 23.4% to 27.6%. The rest of the fall of the aggregate labor share comes after 2024, as the gap stops widening and the rental rate returns to r_*.

Figure 8 places the model paths inside the band of the HP trends of the twelve measures of Figure 2, the trend counterpart of the raw band of Figure 3. From 2004 on the baseline

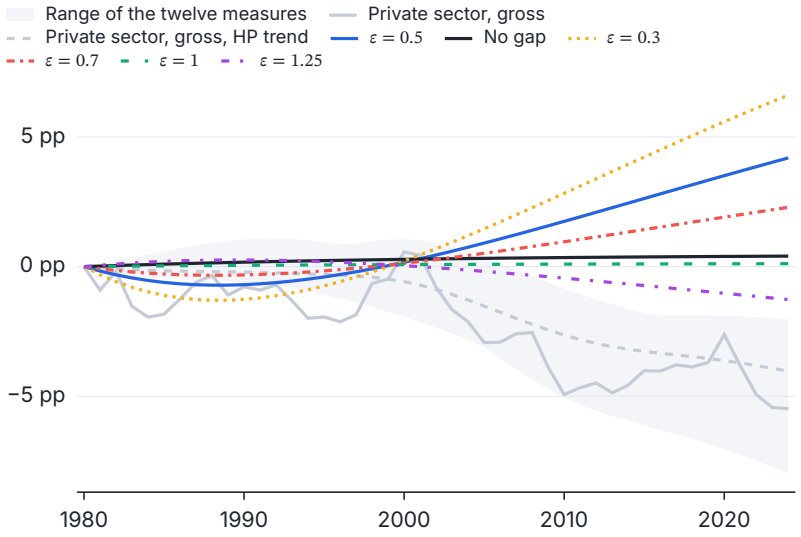
path lies above the highest of the twelve trends, and the other four paths cross that edge between 2003 and 2008. In 2024 the measures are between 3.5 and 10.3 pp below their 1980 values, and their HP trends between 2.0 and 8.0 pp below. The model paths change by −1.3 to 6.6 pp, all above the band. Read against the band, sectoral technology differences are a force in the opposite direction to the measured change. With an elasticity below one the rise is comparable in size to the 4.0 pp decline of the private-sector gross measure, the measure that matches the definition of the model's labor share. Under Cobb-Douglas technology it is small. At $\varepsilon = 1.25$ it turns in the direction of the measured change and remains small. The transition above is a mechanism that predicts a rise, of the kind that Grossman & Oberfield (2022) suggest the literature has set aside (Section 1). Its size depends on the fitted path of the gap and on the elasticity. The relative price is matched equally well at the five elasticities (Figure 6), so the path of the gap is set at each elasticity and the elasticity comes from the estimates of Section 3.5. Their range, 0.3 to 0.7, maps into a rise of 2.3 to 6.6 pp.

4.3. Robustness

Figure 9 and Figure 10 repeat the baseline at $\varepsilon = 0.5$ under five changes to choices that the main exercise holds fixed. A sixth change, reported in the text only, lowers the Frisch elasticity to 0.25. Each case refits the profile of the growth rate of the gap to the relative price and keeps the calibration rules of Section 4.1. The economy starts from its 1980 state in every case, with $\tilde{k}_0$ and A_I re-solved so that $\tilde{k}_{C0} = 1$ and the 1980 capital-output ratio hold, and the change in the labor share is read next to that of the constant-gap path.

Figure 9 varies preferences and the common growth rate. In the baseline, consumption per household grows at 2.46% a year over 1980–2024 in the model and at 1.77% in the data, because the widening gap raises the growth of consumption above g. In the case with g

Figure 8. The labor share of the model against the band of measures: change since 1980.

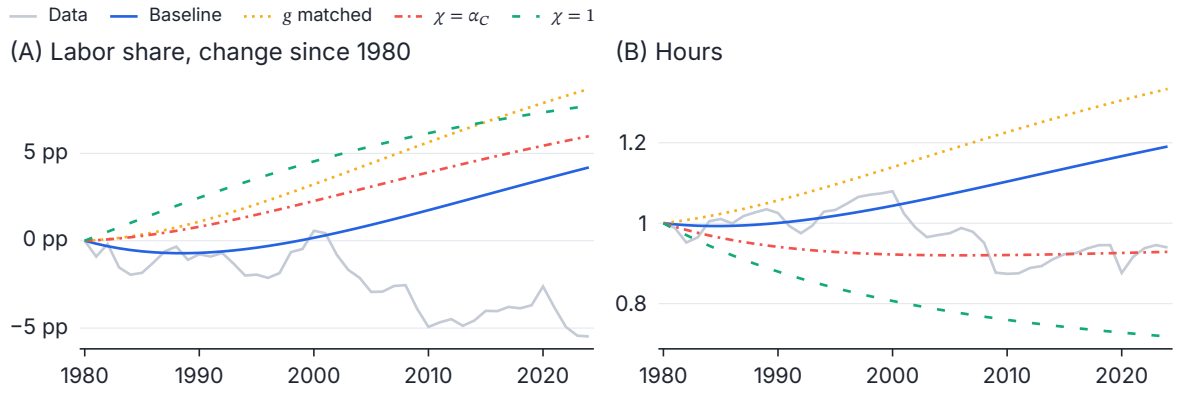


Notes: Each series minus its 1980 value, in percentage points. Shaded area: the range of the HP trends ($\lambda = 400$) of the twelve measures of Figure 2, each minus its 1980 trend value. Gray lines: the private-sector gross measure and the change of its HP trend. Colored lines: the labor share of the model, one path per elasticity of substitution. Black line: the constant-gap path at the baseline elasticity.

matched, $g = 0.52\%$ is the value at which the two growth rates coincide. A lower g lowers the rental rate of the balanced growth path and raises its capital intensity, while the 1980 state keeps the capital-output ratio of the data. The economy therefore starts further below its balanced growth path and accumulates capital along the transition. The labor share rises by 8.7 pp. The convergence accounts for 5.0 pp, against 0.4 pp in the baseline. The effect of the widening gap is 3.7 pp, close to the 3.8 pp of the baseline. Hours rise to 1.33 times their 1980 level. With a Frisch elasticity of 0.25, the value of Elminejad *et al.* (2023), hours rise to 1.07 times their 1980 level, against 1.19 in the baseline. The labor share rises by 5.3 pp, and the effect of the widening gap is 4.9 pp. The other two cases change the trend in the disutility of hours to $\Gamma_t = A_C z_{Ct} (\zeta_t / \zeta_0)^{\chi}$, with $\chi \geq 0$ a parameter. This trend grows at g on every balanced growth path, and $\chi = 0$ is Equation 29. With $\chi = \alpha_C$ the trend is a constant multiple of the scaling of Proposition 4 under Cobb-Douglas technology, $A_C z_{It}^{\alpha_C} z_{Ct}^{1-\alpha_C}$. With it the trend grows with consumption per household and hours are stationary for any path of the gap. With $\chi = \alpha_C$ hours fall to 0.93 times their 1980 level, against 0.92 in the data, and the labor share rises by 6.0 pp. With $\chi = 1$, hours fall to 0.72 and the labor share rises by 7.7 pp. The rise in hours of the baseline therefore depends on $\chi = 0$.

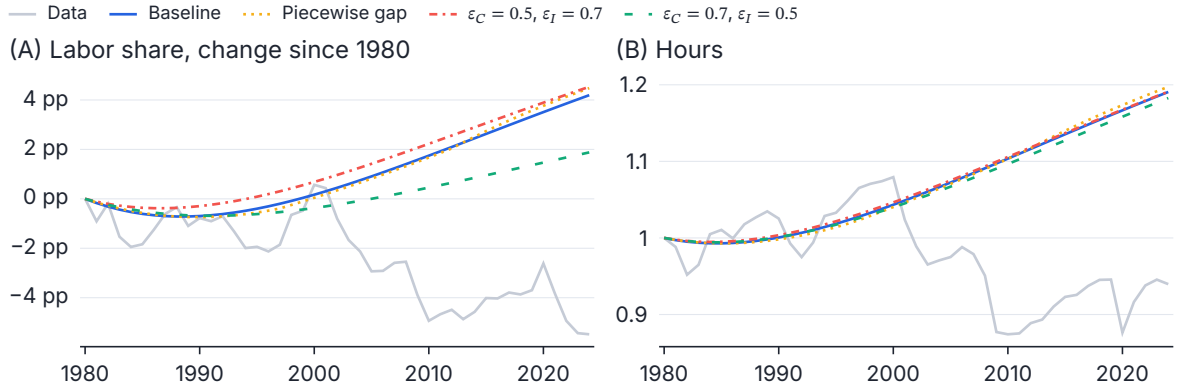
Figure 10 varies the technology. The first case holds a growth rate of the gap constant within each phase of the relative price, 1980–1997, 1997–2012 and 2012–2024, and sets it to zero from 2024 on. It fits the relative price with an error of 1.3 log points, against 1.7 for the exponential profile. Its three values are 2.8%, 1.8%, and 0.6% a year. The labor share rises by 4.5 pp, and the BGP labor share at the end of the transition is 60.9%, against 64.1% in the baseline. With $\varepsilon_C = 0.5$ and $\varepsilon_I = 0.7$ the labor share rises by 4.5 pp, close to the 4.2 pp of the baseline. With $\varepsilon_C = 0.7$ and $\varepsilon_I = 0.5$ it rises by 1.9 pp, close to the 2.3 pp of the case with both elasticities at 0.7.

Figure 9. Robustness to preferences and the common growth rate.



Notes: Annual, 1980–2024. Each case refits the profile of the growth rate of the gap to the relative price. The data in gray. The baseline at $\varepsilon = 0.5$, the common rate g matched to the growth of consumption per household, and the trend $\Gamma_t = A_C z_{Ct} (\zeta_t / \zeta_0)^{\chi}$ with $\chi = \alpha_C$ and $\chi = 1$. Labor share: private-sector gross measure of Figure 2, minus its 1980 value. Hours: BLS hours over the number of households, 1980 = 1.

Figure 10. Robustness to the profile of the growth rate of the gap and the elasticities of substitution.



Notes: Annual, 1980–2024. Each case refits the profile of the growth rate of the gap to the relative price. The data in gray. The baseline at $\varepsilon = 0.5$, a growth rate of the gap constant within each phase of the relative price and zero from 2024 on, and unequal elasticities. Data as in Figure 9.

5. Conclusions

This paper tries to answer how technology differences between the consumption and the investment sector move the labor share when capital and labor are gross complements. The answer has a theoretical and a quantitative part.

When capital and labor are complements, the share of a factor rises with its relative cost, because firms substitute away from it by less than the cost rises. The wage is common to the two sectors and grows with the faster technology of the investment sector. In the consumption sector it therefore grows faster than the sector's own technology, labor becomes more costly per efficiency unit, and its share of the sector's income rises. Capital moves toward the investment sector, whose labor share is the higher of the two at the base point. The composition of output adds to the rise while that ordering holds. Under Cobb-Douglas technology the sector shares are fixed and only the composition effect remains. Above one the within-sector effect changes sign. Along the balanced growth path the relative price and the labor share are constant, at any elasticity of substitution. If the gap stops widening, the effect of sectoral technology differences on the labor share is a transition effect plus a level effect, through the level of the gap at which the transition ends.

The growth rate of the technology gap is fitted to the relative price of consumption to investment over 1980–2024 and to nothing else. The fit puts it at 3.4% a year in 1980, halving every 23 years. The economy starts from its 1980 state, so the change below combines the convergence toward the balanced growth path with the effect of the widening gap. The constant-gap path puts the convergence at 0.4 pp at the baseline elasticity, and the rest is the widening gap. Under the fitted gap the labor share rises by 4.2 pp at the baseline elasticity of 0.5. It rises by 2.3 to 6.6 pp over the range of the firm- and industry-level estimates, and by 0.1 pp in the Cobb-Douglas case. As in Karabarbounis & Neiman (2014), an elasticity above one leads to a decline of the labor share. At their estimate of 1.25 it falls by 1.3 pp in this model, less than the measured decline. The measured share falls over the same years, by 2.0 to 8.0 pp across twelve measures, and the baseline path leaves the band of measures in

2004. The relative price determines the path of the gap at each elasticity, and the elasticity comes from the firm- and industry-level estimates. Every value in their range gives a rise. Until 2000 the model's share and the measured one change little (0.2 and −0.6 pp). The two part over 2000–2012, when the measures fall by 2.0 to 4.4 pp and the model's share rises by 1.9 pp. They diverge further after 2012.

In the model capital moves toward the investment sector, hours per household rise, and output, consumption, and the wage grow faster than in the data. The wage runs ahead from the start, output and consumption from the early 2000s. In the data capital moves away from the investment sector, and hours fall. The rise in hours depends on the trend in the disutility of labor. With a trend that also grows with the gap, hours fall in the model as well (Section 4.3). The technology gap that the relative price requires thus moves more variables than the aggregate share against the data.

Three simplifications qualify these numbers. The depreciation rate is constant. In the data it rises, mostly through higher rates within asset types and partly through the shift of the capital stock toward intellectual property products. The gross labor share of the model inherits the simplification. The growth rate of the gap decays along a single exponential, which reproduces the slowdown of the relative price after 2012 only in part. The model adds 7 log points over 2012–2024 where the data add 3. A growth rate of the gap constant within each phase of the relative price gives a rise of 4.5 pp, and unequal elasticities in the two sectors give 1.9 to 4.5 pp (Section 4.3). A Frisch elasticity of 0.25 gives a rise of 5.3 pp, against 4.2 pp in the baseline. Markets are competitive, so the model has none of the nonlabor income beyond the rental rate that the measured return on capital contains.

With an elasticity below one, the technology that the relative price reveals is a force in the opposite direction to the measured decline of the labor share. Its size is comparable to the decline of the private-sector gross measure, the measure that matches the definition of the model's labor share. An account of the decline that keeps this technology has to cover the measured fall and the rise that the technology adds. The wedges that market power introduces between factor prices and marginal products are the candidates examined in Gouveia-Mendes (2026), within the same model.

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A. Appendix

A.1. Proof of Proposition 1

The household's first-order conditions for hours and consumption give the labor supply Equation 4, $w_t = \Gamma_t q_t \ell_t^{\eta}$. Substituting it into the labor condition of sector C in Equation 11, with $\Gamma_t = A_C z_{Ct}$ from Equation 29,

$$q_t A_C (1-\alpha_C) z_{Ct} F_C\big(\tilde{k}_{Ct}\big)^{\frac{1}{\varepsilon_C}} = A_C z_{Ct} q_t \ell_t^{\eta} \Leftrightarrow \ell_t = \left[(1-\alpha_C) F_C\big(\tilde{k}_{Ct}\big)^{\frac{1}{\varepsilon_C}}\right]^{\frac{1}{\eta}},$$

which is Equation 24.

Dividing the capital condition by the labor condition in each sector gives Equation 13, that is,

$$\frac{w_t}{r_t} = z_{jt} \tilde{k}_{jt}^{\frac{1}{\varepsilon_j}} \frac{1-\alpha_j}{\alpha_j}.$$

Since both sectors face the same w_t and r_t, the ratio of the two expressions gives

$$\zeta_t \tilde{k}_{It}^{\frac{1}{\varepsilon_I}} \tilde{k}_{Ct}^{-\frac{1}{\varepsilon_C}} = \frac{\alpha_I}{1-\alpha_I} \frac{1-\alpha_C}{\alpha_C} \equiv \mathcal{A}, \tag{A.1}$$

the equalization of the marginal rates of technical substitution across sectors. Substituting Equation 23 gives Equation 25. The left-hand side of Equation 25 is the product of three factors. The first, $\zeta_t^{1-1/\varepsilon_C} \tilde{k}_t^{1/\varepsilon_I - 1/\varepsilon_C}$, is positive and does not depend on $s_{\ell t}$. The second, $(s_{kt}/s_{\ell t})^{1/\varepsilon_I}$, decreases in $s_{\ell t}$ from $+\infty$ to $(s_{kt})^{1/\varepsilon_I}$. The third, $((1-s_{\ell t})/(1-s_{kt}))^{1/\varepsilon_C}$, decreases from $(1-s_{kt})^{-1/\varepsilon_C}$ to 0. Hence it is strictly decreasing in $s_{\ell t}$ with the limits stated, and the root is unique.

The ratio of the two capital conditions in Equation 12 gives Equation 28. Substituting r_{t+1} from the capital condition of sector I, $\tilde{c}_t$ from Equation 5, and

$$\frac{c_t}{\Gamma_t} = (1-s_{\ell t}) \ell_t F_C\big(\tilde{k}_{Ct}\big),$$

from the technology of sector C and market clearing, into the Euler equation gives Equation 26. Finally, capital accumulation,

$$k_{t+1} = (1-\delta) k_t + y_{It}, \quad y_{It} = A_I z_{It} s_{\ell t} \ell_t F_I\big(\tilde{k}_{It}\big),$$

divided by $z_{It}\ell_t$, gives Equation 27. Given the path of the four reduced variables, the remaining objects follow. The sector intensities $\tilde{k}_{jt}$ follow from Equation 23, and the capital stock from $k_t = \tilde{k}_t z_{It}\ell_t$. The sector inputs follow from the shares, outputs from the technologies, and c_t and i_t from market clearing. Prices follow from Equation 28 for q_t, the capital condition of sector I for r_t, and Equation 4 for w_t.

A.2. Proof of Proposition 2

With all reduced variables constant and $\zeta_t = \zeta_*$, q_t is constant by Equation 28, and Equation 26 reduces to

$$1 = \beta_\Gamma \frac{1-\delta+r_*}{\Gamma_*}, \quad r_* = A_I \alpha_I \left(\frac{F_I(\tilde{k}_{I*})}{\tilde{k}_{I*}}\right)^{\frac{1}{\varepsilon_I}}.$$

Since $\Gamma_* = 1+g$ and $\beta_\Gamma = \beta(1+g)^{1-\varphi}$, this gives Equation 30. Inverting the capital condition of sector I for $\tilde{k}_{I*}$, with $F_I(\tilde{k})/\tilde{k} = \left[\alpha_I + (1-\alpha_I)\tilde{k}^{-\rho_I}\right]^{1/\rho_I}$,

$$\left[\alpha_I + (1-\alpha_I)\tilde{k}_{I*}^{-\rho_I}\right]^{\frac{1}{\rho_I}} = \left(\frac{\bar{r}_*}{\alpha_I}\right)^{\varepsilon_I} \Leftrightarrow \tilde{k}_{I*}^{-\rho_I} = \frac{\left(\bar{r}_*/\alpha_I\right)^{\varepsilon_I - 1} - \alpha_I}{1-\alpha_I},$$

which is Equation 31 after raising both sides to $-1/\rho_I = \varepsilon_I/(1-\varepsilon_I)$. The right-hand side is positive if and only if $\left(\bar{r}_*/\alpha_I\right)^{\varepsilon_I - 1} > \alpha_I$, the first condition in Equation 37. For $\varepsilon_I < 1$ it reads

$$r_* < A_I \alpha_I^{\frac{\varepsilon_I}{\varepsilon_I - 1}},$$

the bound in Equation 14. From Equation 27 at the steady state, $(g+\delta)\tilde{k}_* = A_I s_{\ell*} F_I(\tilde{k}_{I*})$. Multiplying by $s_{k*}/s_{\ell*}$ and using $\tilde{k}_{I*} = s_{k*}\tilde{k}_*/s_{\ell*}$,

$$s_{k*} = (g+\delta)\frac{\tilde{k}_{I*}}{A_I F_I(\tilde{k}_{I*})} = \frac{g+\delta}{A_I}\left(\frac{\alpha_I}{\bar{r}_*}\right)^{\varepsilon_I},$$

which is Equation 32, where the last step uses the capital condition of sector I. $s_{k*} < 1$ is the second condition in Equation 37. Solving Equation (A.1) for $\tilde{k}_{C*}$ gives Equation 33. From Equation 23,

$$\tilde{k}_* = \tilde{k}_{I*}\frac{s_{\ell*}}{s_{k*}}, \quad \tilde{k}_{C*} = \frac{1-s_{k*}}{1-s_{\ell*}}\frac{s_{\ell*}}{s_{k*}}\tilde{k}_{I*}\zeta_*.$$

Solving the second for $s_{\ell*}$ gives Equation 34, and the first is Equation 35. Equation 24 at the steady state gives Equation 36. The argument of the utility function is

$$\tilde{c}_* = (1 - s_{\ell *})\ell_* F_C\big(\tilde{k}_{C*}\big) - \frac{\ell_*^{1+\eta}}{1+\eta}.$$

Substituting ℓ_*^{η} from Equation 36 and using $(1-\alpha_C)F_C\big(\tilde{k}\big)^{1/\varepsilon_C - 1} = 1 - \psi_C\big(\tilde{k}\big)$,

$$\tilde{c}_* > 0 \Leftrightarrow (1 - s_{\ell *})(1+\eta) > 1 - \psi_C\big(\tilde{k}_{C*}\big),$$

the third condition in Equation 37. Since the right-hand side is below one, the condition holds whenever $s_{\ell *} < \eta/(1+\eta)$.

A.3. Proof of Corollary 1

With $\zeta_t = \zeta_*$, both technology levels grow at g. Hours are constant at ℓ_*. From $\tilde{k}_t = \tilde{k}_*$,

$$\frac{k_{t+1}}{k_t} = \frac{z_{It+1}}{z_{It}} \frac{\ell_{t+1}}{\ell_t} = 1 + g,$$

and the sector capital stocks grow at the same rate since s_{kt} is constant. From Equation 10 with constant intensities and shares, $y_{jt+1}/y_{jt} = z_{jt+1}/z_{jt} = 1 + g$ in both sectors, hence consumption and investment grow at g by market clearing. q_t is constant by Equation 28, r_t by the capital condition of sector I, and ψ_{jt} by Equation 16 with constant intensities. The wage $w_t = \Gamma_t q_t \ell_t^{\eta}$ grows with Γ_t, at g. The Euler equation with constant $\tilde{c}_t$ and q_t gives

$$1 + r_* - \delta = \frac{\Gamma_*}{\beta_\Gamma} = \frac{(1+g)^{\varphi}}{\beta}.$$

A.4. Proof of Proposition 3

From the firms' conditions Equation 11 and Equation 12, the elasticities of output with respect to labor and capital are

$$\big(1-\alpha_j\big)F_j\big(\tilde{k}\big)^{\frac{1}{\varepsilon_j}-1} = 1 - \psi_j\big(\tilde{k}\big), \quad \alpha_j\left(\frac{F_j\big(\tilde{k}\big)}{\tilde{k}}\right)^{\frac{1}{\varepsilon_j}} \frac{\tilde{k}}{F_j\big(\tilde{k}\big)} = \psi_j\big(\tilde{k}\big),$$

which gives Equation 38 and Equation 39. Dividing the two capital payments gives Equation 40, and dividing y_{It} by y_t in Equation 41 gives ω_t in Equation 42. Labor income is

$$w_t \ell_t = (1 - \psi_{Ct}) q_t y_{Ct} + (1 - \psi_{It}) y_{It}.$$

Dividing by y_t, using Equation 40, and multiplying numerator and denominator by $\psi_{Ct} s_{kt}$ gives Equation 43. The decomposition Equation 44 follows from the definitions of ω_t and of the sector labor shares. For Equation 45, substitute Equation 13 into Equation 16:

$$\alpha_j \tilde{k}_{jt}^{\rho_j} = \alpha_j^{\varepsilon_j}\left(1-\alpha_j\right)^{1-\varepsilon_j}\left(\frac{r_t z_{jt}}{w_t}\right)^{1-\varepsilon_j},$$

and divide numerator and denominator of $1-\psi_j$ by $\left(1-\alpha_j\right)^{1-\varepsilon_j}\left(z_{jt}/w_t\right)^{1-\varepsilon_j}$.

A.5. Proof of Corollary 2

With $\varepsilon_j = 1$, $\rho_j \to 0$ and $F_j\left(\tilde{k}\right) \to \tilde{k}^{\alpha_j}$, so Equation 10 becomes

$$y_{jt} = A_j k_{jt}^{\alpha_j}\left(z_{jt}\ell_{jt}\right)^{1-\alpha_j},$$

and Equation 16 gives $\psi_j\left(\tilde{k}\right) = \alpha_j$ for every $\tilde{k}$. Substituting into Equation 43 gives Equation 48, and into Equation 44 gives $m_{\ell jt} = 1-\alpha_j$.

A.6. Balanced Growth with Unequal Technology Growth Rates

Proposition 2 covers the BGP used in the paper, with equal technology growth rates in the two sectors. This section states what a permanent difference between the growth rates does to balanced growth.

Proposition 4. *(Unequal Growth Rates).* If $\varepsilon_C \neq 1$, a BGP exists only if $g_{z_I} = g_{z_C}$: a permanent difference between the technology growth rates of the two sectors is not compatible with a BGP. If $\varepsilon_C = 1$, a BGP exists for any $\left(g_{z_I}, g_{z_C}\right)$ with the scaling $\Gamma_t = A_C z_{It}^{\alpha_C} z_{Ct}^{1-\alpha_C}$ in place of Equation 29, and along it the relative price grows at the constant rate

$$\frac{q_{t+1}}{q_t} = \left(\frac{1+g_{z_I}}{1+g_{z_C}}\right)^{1-\alpha_C}. \tag{A.2}$$

The restriction is the two-sector form of the theorem of Uzawa (1961). With an elasticity of substitution different from one in the consumption sector, a permanent difference in technology growth between sectors keeps moving the effective capital intensity of the consumption sector. No constant allocation of capital and hours across sectors is consistent with it. With $\varepsilon_C < 1$ and $g_{z_I} > g_{z_C}$, the intensity of the consumption sector grows without bound, by Equation 25, as long as the effective capital intensity of the investment sector stays bounded away from zero. A permanent difference between the technology growth rates is therefore compatible with a BGP only under Cobb-Douglas technology in the consumption sector. There sectoral technology differences change the relative price permanently, at the rate in Equation (A.2). That is the BGP with investment-specific technical change of Greenwood *et al.* (1997). Otherwise the economy still has an equilibrium path under a permanent difference between the growth rates, which the paper does not compute. Along that path, with $\varepsilon_C < 1$ and $g_{z_I} > g_{z_C}$, the capital cost share of the consumption sector tends to zero, by Equation 17.

Proof. At a steady state of the reduced system,

$$\tilde{k}_{It} = \frac{s_{k*}}{s_{\ell*}}\tilde{k}_*, \quad \tilde{k}_{Ct} = \frac{1 - s_{k*}}{1 - s_{\ell*}}\tilde{k}_*\zeta_t,$$

so the intensity of sector I is constant, while that of sector C is constant only if ζ_t is. If $\varepsilon_C \neq 1$ and ζ_t is not constant, the left-hand side of Equation 25 changes with ζ_t at constant $(s_{k*}, s_{\ell*}, \tilde{k}_*)$, since its exponent $1 - 1/\varepsilon_C$ is not zero, and Equation 24 changes with $\tilde{k}_{Ct}$. No constant solution exists. If $\varepsilon_C = 1$, ζ_t enters Equation 25 through $\zeta_t^{1-1/\varepsilon_C} = 1$ and Equation 24 through the factor $\zeta_t^{\alpha_C/\eta}$, since $F_C(\tilde{k})^{1/\varepsilon_C} = \tilde{k}^{\alpha_C}$. With $\Gamma_t = A_C z_{It}^{\alpha_C} z_{Ct}^{1-\alpha_C}$ in place of Equation 29, the factor cancels in the labor supply, and the reduced system is autonomous for any (g_{z_I}, g_{z_C}). For the relative price, Equation 28 with

$$\left(\frac{F_I(\tilde{k}_I)}{\tilde{k}_I}\right)^{\frac{1}{\varepsilon_I}} = \tilde{k}_I^{\alpha_I - 1}, \quad \left(\frac{\tilde{k}_C}{F_C(\tilde{k}_C)}\right)^{\frac{1}{\varepsilon_C}} = \tilde{k}_C^{1-\alpha_C},$$

after substituting Equation 23, gives

$$\begin{aligned}\frac{q_{t+1}}{q_t} = &\left(\frac{s_{\ell t+1}}{s_{\ell t}}\frac{s_{kt}}{s_{kt+1}}\right)^{1-\alpha_I}\left(\frac{1 - s_{\ell t}}{1 - s_{\ell t+1}}\frac{1 - s_{kt+1}}{1 - s_{kt}}\right)^{1-\alpha_C} \times \\ &\times \left(\frac{1 + g_{z_I}}{1 + g_{z_C}}\right)^{1-\alpha_C}\left(\frac{\tilde{k}_{t+1}}{\tilde{k}_t}\right)^{\alpha_I - \alpha_C},\end{aligned}$$

and with constant shares and intensity along the BGP only the technology term remains, which is Equation (A.2). □